Sharing Damages Among Multiple Tortfeasors*

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In our society, losses are often caused by the activities of more than one actor. The apportionment of such losses among the various responsible actors raises difficult and controversial questions. Recently, courts have confronted these questions primarily when determining whether defendants held jointly and severally liable for a given harm have the right to seek contribution from other responsible parties. The availability of a right to contribution received particular attention in the early 1980s when the United States Supreme Court decided whether the right existed under antitrust law1 and employment discrimination law.2 More recently, the

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availability of contribution has been the focus of judicial attention in such
dissimilar areas as securities law and environmental law.

Not surprisingly, these cases attracted the attention of the academic
literature, in particular the law-and-economics literature, to the choice be-
tween rules of contribution and no contribution. Several commentators,
most importantly William Landes and Richard Posner, writing shortly
before the two Supreme Court decisions, asked whether apportioning a
loss among the responsible parties through contribution was more desira-
ble than the traditional common law rule that barred such contribution. They concluded that as long as the relevant actors are risk-neutral, no-
contribution rules are efficient and provide the same incentives as rules
that apportion the resulting harm.

The narrow focus of these commentators on the effects of a right to
contribution obscured the most important questions raised by the apor-
tionment of losses among responsible parties. First, because there can be
no right to contribution without joint and several liability, the commenta-
tors assumed, without much discussion, that rules of joint and several lia-

3. Most recently, the Supreme Court granted certiorari to consider a contribution claim brought
whether there was a right to contribution. See Pinter v. Dahl, 108 S. Ct. 2063, 2070 n.9 (1988).

4. Most importantly, this question has arisen under the Comprehensive Environmental Response,
which imposes liability in connection with the disposal of hazardous wastes. See, e.g., United States v.
New Castle County, 642 F. Supp. 1258, 1269 (D. Del. 1986); Colorado v. ASARCO, Inc., 608 F.
Supp. 1484, 1491 (D. Colo. 1985). Subsequent to these decisions, Congress explicitly provided a right

5. Landes & Posner, Joint and Multiple Tortfeasors: An Economic Analysis, 9 J. LEGAL STUD.
TORT LAW 190 (1987). Two subsequent articles, which focused on the right to contribution under
the antitrust laws, relied heavily on the analysis of Landes and Posner. See Easterbrook, Landes &
Posner, Contribution Among Antitrust Defendants: A Legal and Economic Analysis, 23 J.L. &
ECON. 331 (1980); Polinsky & Shavell, Contribution and Claim Reduction Among Antitrust Defend-
ants: An Economic Analysis, 33 STAN. L. REV. 447 (1981). There is also a brief, recent discussion in
S. SHAVELL, ECONOMIC ANALYSIS OF ACCIDENT LAW 164–67 (1987). The classic piece on joint and
several liability and contribution is Leflar, Contribution and Indemnity Between Tortfeasors, 81 U.
PA. L. REV. 130 (1932).

One recent study of joint and several liability was published after we had completed our work on
this Article. See Wright, Allocating Liability Among Multiple Responsible Causes: A Principled De-
fense of Joint and Several Liability for Actual Harm and Joint Exposure, 21 U.C. DAVIS L. REV.
1141 (1988). Its discussion of the efficiency considerations raised by the allocation of losses among
joint tortfeasors does not expand upon the theoretical work of Landes & Posner, supra. See Wright,
supra, at 1175–76.

6. See supra text accompanying notes 1–2.

7. The no-contribution rule of the common law is generally traced to Merryweather v. Nixan, 8
this rule. See infra note 48.

8. See Landes & Posner, supra note 5, at 521–24. A risk-neutral actor is indifferent between
between accepting a gamble and receiving the expected payoff of that gamble. Thus, a risk-neutral
actor is indifferent between receiving $10 with certainty and a 10% chance of receiving $100 (and a
90% chance of receiving nothing). In contrast, a risk-averse actor would prefer to receive the expected
payoff, and a risk-prefering actor would prefer to gamble.

For discussions of whether corporations are risk-neutral, see K. ELZINGA & W. BREIT, THE ANTI-
TRUST PENALTIES 126–29 (1976); Easterbrook, Landes & Posner, supra note 5, at 351–52 & n.50;
Polinsky & Shavell, supra note 5, at 452–55.
bility would govern. In so doing, they did not consider the properties of several classes of apportionment rules not predicated on a finding of joint and several liability, and did not provide any normative guidance as to when such liability is desirable from an efficiency perspective.

Second, these commentators considered the choice between apportionment and no-contribution only under negligence, and did not examine this choice under strict liability. They shed no light on the efficiency implications, in the context of multiple tortfeasors, of the shift from negligence to strict liability in many areas of accident law, and of the adoption of strict liability under important federal statutes.

In this Article, we show that many different classes of rules of apportionment are being applied by common law courts and advocated by commentators. The question of which rules to apply in particular situations is fraught with confusion.

We then examine the varying efficiency properties of these rules under negligence. In addition, we demonstrate that the efficiency of apportionment rules under negligence depends almost exclusively on whether the negligent actors must pay for damage caused by non-negligent actors, as they must under joint and several liability. Except where one actor's damage does not affect the extent of damage caused by another, only joint and several liability rules produce the efficient result. We use this insight to provide an economic reinterpretation of the terms “distinct,” “divisible,” and “indivisible” harms, which determine, both in the Restatement (Second) of Torts and in common law cases, whether joint and several liability should attach.

We then consider questions of apportionment under strict liability. All the rules of apportionment currently used by the courts are inefficient. Thus, if courts were limited to a choice of traditional rules of apportionment, efficiency considerations would argue that the rule of liability should be negligence rather than strict liability.

We show, however, that there is a plausible class of apportionment rules under strict liability that produce the efficient outcome. These rules, which attach significance to whether an actor is taking the socially optimal amount of care, combine the distributional consequences of strict liability with the incentives of a negligence regime. We do not believe that

9. We interpret the analyses of contribution in antitrust law as ones involving negligence rules. See Easterbrook, Landes & Posner, supra note 5; Polinsky & Shavell, supra note 5. We do so because each conspirator could avoid liability entirely simply by refusing to join the conspiracy. Refusal to enter the conspiracy is thus equivalent to the decision to meet the standard of care.


11. See infra note 27 (discussing CERCLA).

12. See Restatement, supra note 10, § 433A.
such rules have been applied by the courts or discussed in the secondary literature. Yet, there are no logistical bars to their adoption, as they are no more cumbersome than simple rules of negligence. Under such apportionment rules, we can compensate victims fully for their losses through strict liability, without thereby creating the inefficiencies that attach under traditional strict liability rules. The choice between negligence and strict liability, therefore, need not, as it does now, give rise to a trade-off between efficiency and distributional concerns.\textsuperscript{13}

We then consider how, given joint and several liability, contribution rules compare to no-contribution rules—the question that was the focus of attention in the early 1980s.\textsuperscript{14} For negligence, our results are consistent with those of the prior literature.\textsuperscript{15} For strict liability, however, no-contribution rules are generally inefficient and fail to create the same incentives as contribution rules; thus, the insights of this literature are not applicable outside of negligence.

Finally, we discuss how the various common law rules perform under negligence when courts err in defining the standard of care. Here, too, we analyze the relative merits of joint and several liability.

This Article’s conclusion that several common law rules of apportionment are not efficient has more than merely descriptive significance. First, in the absence of overriding goals, the legal system should favor efficient rules; while efficiency might not be the only relevant goal, it clearly is a cognizable one.\textsuperscript{16} Thus, the efficiency properties of joint and several liability should be considered in evaluating the recent efforts by several state legislatures to limit joint and several liability in the name of tort reform.\textsuperscript{17} Second, the Supreme Court’s current interest in economic analysis suggests that the development of the common law, or at least of the federal

\textsuperscript{13} Traditional strict liability rules might still be preferred over both negligence and the strict liability rules that we propose on the ground that they are easier to administer. See infra text accompanying note 99.

\textsuperscript{14} In the absence of joint and several liability, the question does not arise, because only an actor who has paid more than her equitable share of the liability can have a right to contribution. See RESTATEMENT, supra note 10, § 886A(2).

\textsuperscript{15} See supra text accompanying notes 5–8.


common law, will be affected by the efficiency properties of competing legal rules.

Section I describes the model implicit in the remainder of our analysis. Section II defines the classes of apportionment rules currently employed by courts. Sections III and IV discuss the efficiency of such rules under negligence and strict liability, respectively. Section V discusses the efficiency of no-contribution rules under both negligence and strict liability, and compares it with that of rules of apportionment. Section VI analyzes the effects of departures from the social optimum in the determination of the standard of care under negligence. The Appendix provides formal proofs for various propositions.

I. DESCRIPTION OF THE MODEL

In this Section, we describe the formal model set forth in the Appendix, which provides the basis for our analysis. Because our work was initially motivated by questions raised by the Comprehensive Environmental Response, Compensation and Liability Act (CERCLA) and because this Article is part of a larger work concerning the incentives generated by the statute, this description, and most of our examples, concern the disposal of hazardous wastes. The analysis, of course, applies more broadly.

Imagine a situation in which several manufacturers dump their wastes at a single landfill. The actors benefit from this dumping because the wastes are the by-product of profitable economic activity. At some time in the future, these wastes may leak into the environment and cause serious damage, including, perhaps, the contamination of groundwater supplies. For ease of exposition, we think of this damage as the cost of cleaning up the landfill and the surrounding area affected by the release.

The expected damage of a release is a “social” loss because it does not fall directly on the dumpers unless there is a legal provision shifting the liability to them. Instead, it falls on the victims who would have legal


21. We take the loss to be an impersonal or economic one: restoring the victim to his pre-accident wealth restores him to his pre-accident utility. Following the prior literature on the allocation of damages among parties, see sources cited supra note 5, we do not consider personal injuries. There is, however, a limited literature that considers such injuries in the single-actor context. See, e.g., Cook & Graham, The Demand for Insurance and Protection: The Case of Irreplaceable Commodities, 91 Q.J. ECON. 143 (1977); Note, An Economic Analysis of Tort Damages for Wrongful Death, 60 N.Y.U. L. REV. 1113 (1985).
responsibility for the cleanup, or, alternatively, who would suffer the consequences were the problem left unattended. In our model, each dumper chooses the amount of waste that she will dump; a larger amount will produce a larger benefit to the actor but will also increase the social loss.

The efficient amount of waste is that which maximizes the social objective function: the sum of the benefits derived by the actors minus the social loss. An economically rational dumper, however, does not make this decision based on the social objective function. Instead, she seeks to maximize her private objective function: the benefit that she derives from the activity that leads to the production of the waste minus whatever share of the social loss the legal regime allocates to her.

The share of the social loss borne by each dumper depends both on the liability rule and on the apportionment rule used to divide the social loss among the joint tortfeasors. We consider two liability rules: negligence and strict liability. Negligence rules define a standard of care; if a dumper meets her standard of care, she will bear no portion of the social loss. Under traditional strict liability rules, in contrast, a dumper will bear some portion of the social loss regardless of her level of care. Thus, a strict liability rule can be thought of as the negligence rule that sets the standard of care at zero, so that the standard is always violated.

The second factor that determines each dumper's share of the social loss is the apportionment rule. Because any portion of the social loss that is not borne by the dumpers falls on the victim, this rule determines not only how the joint tortfeasors divide the social loss among themselves but also

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22. Under CERCLA, the victim is often the federal government, which will undertake cleanups if "there may be an imminent and substantial endangerment to the public health or welfare or the environment because of an actual or threatened release of a hazardous substance from a facility." 42 U.S.C. § 9606(a) (1982).

23. We assume that the probability of a release does not depend on the amount dumped, but that, given a release, the damage is a function of this amount.

24. To isolate the issue of apportionment among injurers, we assume that the victim cannot affect the size of that loss. Thus, we deal only with simple negligence rules (rather than negligence rules with contributory negligence) and with simple strict liability rules (rather than strict liability rules with contributory negligence or with dual contributory negligence). See infra note 135 (defining these rules). Our analysis, however, can easily extend to the case in which the victim's behavior affects the size of the loss. Indeed, as we note in the conclusion of this Article, the formal model of this phenomenon is similar to the model that we use.

25. In general, as the derivation of the conditions for the social optimum in the Appendix makes clear, if each actor obtains different benefits from producing the waste, each actor should also face a different standard of care. This result is consistent with Judge Learned Hand's famous formula in United States v. Carroll Towing Co., 159 F.2d 169 (2d Cir. 1947), under which actors with different costs of taking care face different standards of care. See id. at 173.

26. In Section IV.B., however, we propose strict liability rules under which an actor who dumps less than the socially optimal amount might not bear any portion of the loss.

27. Even though CERCLA is a strict liability statute, see, e.g., New York v. Shore Realty Corp., 759 F.2d 1032, 1042 (2d Cir. 1985); City of Philadelphia v. Stepan Chem. Co., 544 F. Supp. 1135, 1140 n.4 (E.D. Pa. 1982), we analyze the effects of apportionment rules under both negligence and strict liability because there is value in thinking about whether the choice of strict liability was wise. Moreover, our model, with suitable reinterpretation for the options available to the actors, is equally applicable to the areas of law in which contribution questions are important. See supra text accompanying notes 1–4.
the victim's share of the damage, which is the total damage minus the sum of the shares of the total damage paid by the dumpers.

Under all eight classes of apportionment rules defined in Section II, the victim's maximum recovery is the total social loss. We exclude the possibility of punitive damages, under which a victim could recover more than the damage that he had suffered, because such damages are generally not available in the absence of intentional wrongdoing.28

From an efficiency perspective, the objective of the liability and apportionment rules is to induce rational actors to produce the socially efficient amount of waste.29 Efficient apportionment rules give each private actor incentives that match those of the social objective function. This Article seeks to define the conditions under which apportionment rules will produce such a match.

II. Rules of Apportionment

We provide a taxonomy that classifies the apportionment rules commonly used by courts and advocated by commentators. The resulting categories are defined by reference to three factors: whether the rules are full liability or partial liability, unitary share or fractional share, and fixed share or proportional share. We end with a simple numerical example that illustrates the operation of the various rules.

The taxonomy that we set forth provides a framework for analyzing the efficiency properties of the various rules. It also isolates those apportionment choices open to the courts that have efficiency consequences from those that do not. Thus, it informs the discussion of the relative merits of the different rules.

A. Full Liability versus Partial Liability

In the single tortfeasor context, most of the literature assumes that the negligent actor is liable for the full loss that is caused by her actions.30 We
shall call such a rule a "full liability" rule. Alternatively, the negligent actor might be responsible for less than the full damage that she causes. We shall refer to such a rule as a "partial liability" rule.

Statutes that limit the liability of actors, such as the $560,000,000 limit of liability for losses from nuclear disasters,\textsuperscript{31} or limits on malpractice liability,\textsuperscript{32} are perhaps the simplest examples of partial liability rules.\textsuperscript{33} Statutory caps on liability, however, create incentives analogous to those that arise when a tortfeasor is insufficiently solvent to pay for her share of the social loss; we discuss these problems in a subsequent study.\textsuperscript{34} In this Article, instead, we consider a second class of partial liability rules, in which a negligent actor is liable only for those losses that would have been prevented through due care, or, stated differently, for the losses that are caused by the actor's negligence. The victim therefore bears the damage that would have resulted even if none of the actors had been negligent. Such a rule is consistent with certain interpretations of the common law of negligence.\textsuperscript{35} For the remainder of this Article, our discussion of partial liability rules is restricted to rules of this type.

Consider, for example, an actor who builds a ninety-foot dam when the standard of care is one hundred feet. Under a partial liability rule, the actor is responsible only for the losses caused by those floods that would have been prevented by a hundred-foot dam; she would not have to pay for the losses caused by floods that would have occurred even if the dam had been one hundred feet high. In contrast, under a full liability rule, the actor is liable for the damages caused by all floods, even those that could not have been avoided if the dam had been a hundred feet high.\textsuperscript{36}

\textsuperscript{33} Such caps, particularly on non-economic damages, are one of the major elements of the recent tort reform movement. See Note, supra note 17, at 628, 637-38. For statutory caps on such damages, see, e.g., FLA. STAT. ANN. § 768.80 (West Supp. 1988) ($450,000); MD. CTS. & JUD. PROC. CODE ANN. § 11-108 (Supp. 1987) ($350,000); N.H. REV. STAT. ANN. § 508:4-d (Supp. 1987) ($750,000).
\textsuperscript{34} Several states have struck down caps on damages as inconsistent with their state constitutions. See, e.g., Kansas Malpractice Victims Coalition v. Bell, 243 Kan. 333, 757 P.2d 251 (1988); Lucas v. United States, 757 S.W. 2d 687 (Tex. 1988). In other states, however, such caps have been upheld. See, e.g., Fein v. Permanente Medical Group, 38 Cal. 3d 137, 157-64, 695 P.2d 665, 679-84, 211 Cal. Rptr. 368, 382-87 (1985); Johnson v. St. Vincent Hosp., 273 Ind. 374, 397-400, 404 N.E.2d 585, 600-01 (1980). For a recent overview of challenges to tort-reform laws, see Barrett, Tort Reform Fight Shifts to State Courts, Wall St. J., Sept. 19, 1988, at B1.
\textsuperscript{35} See infra text accompanying notes 38-39 (discussing whether courts use full liability or partial liability rules).
\textsuperscript{36} We assume that if a flood occurs, the level of damage is independent of the height of the dam.
Under a partial liability rule, whenever a flood occurs a court must inquire whether that flood would have occurred had the defendant not been negligent. If the flood would not have occurred, the defendant must pay the full damages caused by the flood. In contrast, if the flood would have occurred anyway, the defendant would not have to pay any damages at all.37

Commentators are split on whether courts apply a full liability or a partial liability rule. Mark Grady argues that, under what we call in this Article a partial liability rule, a negligent actor could advance the defense that the loss would have occurred even if she had not been negligent. He observes that this defense is quite uncommon, and therefore that courts cannot be applying a partial liability rule.38 In contrast, Marcel Kahan

For similar examples, see Grady, A New Positive Economic Theory of Negligence, 92 Yale L.J. 799, 806-08, 822 (1983); Kahan, supra note 30.

37. In some contexts, one cannot determine with certainty whether the accident would have occurred without negligence. Under such stochastic conditions, the interpretation of a partial liability rule is somewhat more complex. Assume that a negligent driver has a 60% probability of hitting one pedestrian on a particular stretch of road (and a 40% probability of not hitting anyone). If he is not negligent, the driver has a 30% probability of hitting one pedestrian (and a 70% probability of not hitting anyone). The damages suffered by an injured pedestrian are $1000. When a pedestrian is hit by a negligent driver, what portion of this $1000 should be borne by the driver under a partial liability rule?

At first glance, $700 might appear to be the correct amount, since it is the full damage of $1000, discounted by the expected damage of $300 (30% of $1000) that would be caused in the absence of negligence. However, the driver's negligence cannot be said to have caused $700 in damages. The expected damage, given the driver's negligence, is $600 (60% of $1000). The negligence therefore increases the expected damage by $300 ($600 minus $300). Should the negligent driver therefore pay the victim only $300?

If under a partial liability rule the negligent driver were responsible for a damage of only $300, the driver would not pay for the full increase in expected damage caused by his negligence; if an accident occurred, the driver would pay $300, but if an accident did not occur, the driver would pay nothing. Given the 60% probability of an accident occurring when the driver is negligent, the expected cost of the driver's negligence would therefore be only $180 (as compared with an expected damage of $300). For the driver to have sufficient incentive to meet the standard of care and to avoid negligent driving, he would have to pay $500 in the event of an accident, so that his expected cost would be $300 (60% of $500).

More generally, suppose both that every accident causes a fixed loss L, and that the level of carelessness x (a higher x denotes less care) determines the probability that an accident will occur. Let c be the standard of care and suppose that the actor negligently adopts some x > c. We may then calculate A(x), the amount recoverable by a victim, as:

\[ A(x) = \frac{(p(x) - p(c))L}{p(x)} \text{, where } x > c \]

\[ (A(x) = 0, \text{ where } x \leq c). \]


One reason courts might not discount the damage suffered by an injured plaintiff—to take into account the expected damage if the defendant had not been negligent—is that courts may assume that this latter amount is zero. Thus, in the example presented here, the implicit assumption might be that without negligence the probability of hitting a pedestrian would be zero, or, at most, a de minimis value. This type of assumption may well be inappropriate in cases involving exposure to carcinogens, which might involve significant background risk. For one approach to this problem, see Robinson, Probabilistic Causation and Compensation for Tortious Risk, 14 J. Legal Stud. 779 (1985) (courts should award damages based on probability of injury, rather than on injury itself).

38. See Grady, supra note 36, at 822. He believes, however, that courts apply, and should apply, a different causation test in which they compare the costs and benefits of undertaken precautions. See id. at 814-17, 824-29. In an article published after their study of multiple tortfeasors, Landes and Pos-
The Yale Law Journal contends that the common law has, in fact, adopted a partial liability rule. But even if Grady's observation is correct, the partial liability rule still merits study because strong normative arguments have been made in its favor. Kahan argues that this rule is compelled by the requirement of causation: that an injurer's negligence, as opposed to her mere action, has caused the loss. Under the full liability rule, in contrast, the injurer is liable for losses that would have occurred even if she had not been negligent.

The distinction between full liability and partial liability rules, which we have introduced for the case of a single tortfeasor, is present also for multiple tortfeasors. Under a full liability rule, the maximum amount for which the negligent actors can be liable in the aggregate (and which the victim can recover) is the full social loss. In contrast, under a partial liability rule, this amount is reduced to take account of the loss that would have occurred even if all of the actors had been non-negligent. While the law-and-economics literature on joint tortfeasors, including the article by Landes and Posner, has assumed the use of full liability rules, the causation rationale supporting the use of partial liability rules for single actors applies with no less force to multiple tortfeasors.

The distinction between full liability rules and partial liability rules exists only under negligence. It collapses under strict liability because strict liability rules assure that the victim is compensated for his full damage.

See supra note 5, at 228. But even in this model, once a defendant is liable, she must pay for the full damage, with no discount for the expected damage in the absence of negligence. Thus, their model does not deal with the problem of stochastic harms. See supra note 37.

Kahan, supra note 30.

This question is somewhat more complicated where it involves a stochastic harm. But where one cannot say with certainty whether a loss would have occurred in the absence of negligence, the question of causation needs to be reinterpreted. Suppose that a negligent driver causes an accident, and that, without negligence, the probability of this accident would have been thirty percent. In what way, then, can the negligence be said to have "caused" the loss? It did increase the probability of the accident, and therefore the expected value of the loss, but one cannot say, as one could in the dam example above, that the loss would not have occurred without the negligence.

We assume here that all of the damage is caused by negligent parties. We deal below with situations in which the damage is caused both by negligent and non-negligent parties. See infra text accompanying notes 45-47.

As we discuss below, the negligent actors will be liable for this loss under all full liability, unitary share rules. See infra text accompanying note 45. The negligent actors will also be liable for the full social loss under full liability, fractional share rules, provided that all actors contributing to the social loss are negligent. See infra text accompanying note 47.

Landes and Posner assume that if one actor is negligent and the others are not, the negligent actor will pay the full damage, even if that damage, or at least part of that damage, would have occurred even had all parties been non-negligent. Landes & Posner, supra note 5, at 521-23.

This statement holds for the type of partial liability rule that we discuss in the text, under which, for negligence, an actor is liable only for the damage in excess of that which would be caused if all actors were non-negligent. The statement does not apply to strict liability rules with caps. This
B. Unitary Share versus Fractional Share

The law-and-economics literature on joint tortfeasors also assumes that the negligent actors pay not only for the damages attributable to their own actions, but also for the damages attributable to the non-negligent actors. Under a competing rule, the negligent actors do not pay for the damages attributable to the non-negligent actors. We refer to such a rule as a "fractional share" rule because the victim generally recovers only a fraction of the maximum damage to which he would be entitled. Under a fractional share rule, the victim recovers the maximum damage only if all of the actors contributing to the harm are negligent; otherwise, he recovers less than this amount because part of the damage will be attributable to non-negligent actors. The shares of the negligent actors therefore can add up to only a fraction of the maximum damage.

We refer to the rule under which the negligent actors pay for damages attributable to the non-negligent actors as a "unitary share" rule, because under it the victim recovers the maximum loss allowable whenever at least one actor is negligent, regardless of how much of the damage is caused by non-negligent actors. Thus, the shares of the negligent actors are unitary because they add up to the maximum damage recoverable by the victim.

Unitary share rules correspond to rules of joint and several liability with a right to contribution from other negligent tortfeasors, whereas fractional share rules correspond to rules of non-joint (several only) liabil-

\[ p(x) - p(b)L \]

where \( p(b)L \) is the contribution of the background sources. In this example, the factory would pay 60% of the damage caused by each incidence of cancer (50% - 20%/50%).

Indeed, consider the differences between joint and several liability with a right to contribution on the one hand and non-joint liability on the other. First, an actor held jointly and severally liable cannot get contribution for the portion of the damage attributable to the non-negligent actors. Second, such an actor cannot get contribution for the portion of the damage attributable to negligent actors who are insolvent. Third, such an actor must expend the transaction costs necessary to bring the contribution actions. Because in this Article we assume the absence of insolvency and transaction costs, the difference between joint and non-joint liability is who pays for the share of the damage attributable to the non-negligent actors. Under joint liability, this share is borne by the negligent actors, whereas under non-joint liability it is borne by the victim. This is precisely the distinction between unitary and fractional share rules: Joint and several liability with a right to contribution therefore corresponds to a unitary share rule, whereas non-joint liability corresponds to a fractional share rule.

As with the distinction between full liability rules and partial liability rules, the distinction between unitary share and fractional share rules exists only under negligence because, again, strict liability rules compensate the victim for his full damage.

C. Fixed Share versus Proportional Share

The models on joint tortfeasors used in the law-and-economics literature also assume that each negligent actor pays a share of the damage that depends only on the number of actors who are negligent and is independent of the level of care taken by the negligent actors. The most common rule in this family is a per capita rule, under which the amount to be paid by the negligent actors is divided equally among those actors. Thus, in our hazardous waste example, two negligent actors would pay the same amount in damages even though one dumped far more than her standard of care whereas the other violated her standard only by a small margin. Commentators advocate per capita rules because they are easier to implement. Such rules are frequently used to apportion damages under the federal securities laws.

49. For further discussion of these rules, see infra Section III.B.
50. See supra note 29. If transaction costs are positive, joint liability allocates the costs of suing additional defendants to the tortfeasors and non-joint liability allocates them to the victim. Similarly, if some wrongdoers are insolvent, under joint liability other wrongdoers must bear the unfunded portion of their allocable damages. Under non-joint liability, the victim bears this loss.
51. See Landes & Posner, supra note 5, at 522.
52. One may interpret the antitrust analyses of contribution by Easterbrook, Landes & Posner, supra note 5, and Polinsky & Shavell, supra note 5, as using the identical rule. Thus, under these models, the damages resulting from an antitrust conspiracy are divided per capita among the parties that have joined the conspiracy.
One can easily imagine a different set of rules, under which the negligent actors divide their joint liability in proportion to their levels of care. Such rules are now being applied under CERCLA, where courts will apportion liability proportionally to the amount of waste that a defendant has dumped.\textsuperscript{54} We label such rules “proportional share” rules because they allocate damages in proportion to the waste dumped by each of the actors.\textsuperscript{55} In contrast, we refer to rules that apportion damages among the negligent actors without regard to such considerations as “fixed share” rules.\textsuperscript{56}

Because the literature assumed that the negligent actors would pay the portion of the loss attributable to non-negligent actors (that the rules would be unitary share rather than fractional share),\textsuperscript{57} it did not have to allocate any amount of the damage to the non-negligent actors. Under fractional share rules, however, such an allocation is necessary and can be performed either in fixed shares or in proportional shares. Of course, under negligence rules, the non-negligent actors do not actually bear the damage attributable to them in this manner; the victim bears it instead.

The distinction between fixed share and proportional share rules holds under strict liability as well as under negligence. The only necessary reinterpretation is that under traditional strict liability rules, the damage is

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\textsuperscript{54} See Colorado v. ASARCO, Inc., 608 F. Supp. 1484, 1487–88 (D. Colo. 1985). Courts can also consider other factors, such as the toxicity of the hazardous substance involved. Id. These factors are proxies for the risk posed by each generator. Questions of allocation under CERCLA are more complex when they involve generators on the one hand, and transporters of the hazardous wastes or owners of the waste sites on the other, see 42 U.S.C. § 9607(a) (Supp. IV 1986), since the risks posed by these diverse activities are more difficult to compare.

\textsuperscript{55} The proportional share approach is also accepted in Unif. Comparative Fault Act § 2, 12 U.L.A. 43 (1979), which has been adopted by two jurisdictions. For an application of this rule, see, e.g., Celotex Corp. v. Campbell Roofing & Metal Works, Inc., 352 So. 2d 1316 (Miss. 1977); Commercial Union Assurance v. Western Farm Bureau Ins., 93 N.M. 507, 601 P.2d 1203 (1979).

\textsuperscript{56} The categorization scheme outlined in the text is not exhaustive. For example, a different proportional share rule would allocate the damage among the actors in proportion to the amount by which their level of waste exceeds the standard of care, rather than by reference to the actual levels of care.

In antitrust law, damages might be apportioned by market share and thus might depend upon decisions of the actors. Such damages would not depend, however, on the decision that precipitates the liability, which is the decision to conspire.

\textsuperscript{57} See supra text accompanying note 45.
divided among all actors who caused it, rather than simply among the negligent actors.\(^5\)

D. Illustrating the Effects of the Rules

We have thus identified eight categories of apportionment rules under negligence and two categories of apportionment rules under strict liability. To illustrate further the operation of the various rules, we present a hypothetical problem. Consider, once again, the hazardous waste example, in which three waste generators, \(A_1\), \(A_2\), and \(A_3\), operating under a negligence rule, dump hazardous wastes at a single site. The standard of care for each is 10 units. Assume that \(A_3\) meets this standard of care by dumping 10 units, but that \(A_1\) and \(A_2\) do not, dumping 15 and 25 units, respectively; the total amount dumped is therefore 50 units. Assume that the damage resulting from this dumping in the event of a release\(^5\) is the square of the total amount dumped—$2500 in this example.\(^6\) Assume, also, that some time after the dumping a release occurs.

Because \(A_3\) meets the standard of care, she will not be liable for any portion of this damage. The question is how each of the apportionment rules allocates the $2500 among \(A_1\), \(A_2\), and the victim, \(A_0\), which, as the residual bearer of cleanup costs, will have to pay for the damage not covered by \(A_1\) and \(A_2\).

The allocations under the eight rules are indicated in the two 2x2 matrices of Figure I.\(^61\) Under Rules 1 through 4, which are full liability rules, the negligent actors would at most pay the full damage of $2500. Under Rules 1 and 2, which are unitary share rules, \(A_1\) and \(A_2\) pay collectively this full amount and \(A_0\) bears no residual damage. Rule 1 allocates the $2500 equally between \(A_1\) and \(A_2\); each therefore pays $1250 (1/2 of $2500); Rule 2 allocates it proportionally to the amounts dumped by \(A_1\) and \(A_2\); \(A_1\) therefore pays $937 (15/40 of $2500) and \(A_2\) pays $1563 (25/40 of $2500).

Under Rules 3 and 4, which are fractional share rules, \(A_1\) and \(A_2\) do not pay for the portion of the damage attributable to \(A_3\). Rule 3 allocates equal shares to \(A_1\), \(A_2\), and \(A_3\); \(A_1\) and \(A_2\) each therefore pays $833 (1/3 of $2500), and \(A_0\) bears the residual damage of $833 attributable to \(A_3\). Rule 4, instead, allocates shares in proportion to the amounts dumped by \(A_1\), \(A_2\), and \(A_3\). \(A_1\), therefore, pays $750 (15/50 of $2500); \(A_2\) pays $1250...
(25/50 of $2500), and \(A_0\) bears the residual loss of $500 attributable to \(A_3\).

Under Rules 5 through 8, which are partial liability rules, the maximum amount that the negligent actors would pay is $1600: the full damage of $2500 minus the damage that would have occurred if all the actors had been non-negligent, which is $900 (the square of the 30 units that would have been dumped if all three actors had been non-negligent). Under Rules 5 and 6, which are unitary share rules, \(A_1\) and \(A_2\) pay collectively the full $1600 and \(A_0\) bears the residual loss of $900. Rule 5 allocates the $1600 equally between \(A_1\) and \(A_2\); each therefore pays $800 (1/2 of $1600). Rule 6 allocates it proportionally to the amounts dumped by \(A_1\) and \(A_2\); \(A_1\) therefore pays $600 (15/40 of $1600) and \(A_2\) pays $1000 (25/40 of $1600).

Under Rules 7 and 8, which are fractional share rules, \(A_1\) and \(A_2\) do not pay for the portion of the damage attributable to \(A_3\). Rule 7 allocates equal shares of the $1600 to \(A_1\), \(A_2\), and \(A_3\).\(^{62}\) \(A_1\) and \(A_2\) each therefore pays $533 (1/3 of $1600) and \(A_0\) bears the residual loss of $1433. Rule 8, instead, allocates shares of the $1600 in proportion to the amounts dumped by \(A_1\), \(A_2\), and \(A_3\). \(A_1\) therefore pays $480 (15/50 of $1600), \(A_2\) pays $800 (25/50 of $1600), and \(A_0\) bears the residual loss of $1220.\(^{63}\)

Figure 1 illustrates a wide discrepancy in how the different rules allocate the damage. For example, the victim \(A_0\) pays nothing under Rules 1 and 2, but pays more than half the damage under Rule 7. Similarly, \(A_2\) pays almost two-thirds of the damage under Rule 2, but less than one-quarter under Rule 7. It is a powerful reflection of the common law's disarray in this area that even this very simple problem could generate such divergent results, and that each of these results would have some claim to legitimacy.\(^{64}\)

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\(^{62}\) The allocation of an amount of damages to \(A_3\) is not inconsistent with a partial liability rule, which reduces the full damage award by the amount of damage that would have occurred if all actors had been non-negligent. If \(A_3\) had not dumped at all, the total amount dumped would have been 40 units and the amount dumped if the two remaining actors had been non-negligent would have been 20 units. Under a partial liability rule, \(A_1\) and \(A_2\) would pay $1200 (the square of 40 minus the square of 20). Thus, if none of the $1600 were allocated to \(A_3\) in our example, \(A_1\) and \(A_2\) would be responsible for $400 more than the damage they would have caused if \(A_3\) had not dumped at all. In contrast, under Rule 7, which allocates a share to \(A_3\), \(A_1\) and \(A_2\) pay the total of $1066—$134 less than they would have paid if \(A_3\) had not dumped at all. Under Rule 8, they pay $1280—$80 more than they would have paid. Thus, these two rules can be seen as moving \(A_1\) and \(A_2\) closer to where they would have been without \(A_3\).

We do not argue that the only sensible rules are ones that allocate a portion of the $1600 to \(A_3\). Under Rules 5 and 6, for example, no such allocation is made. Rather, we make the narrower claim that there is a principled basis for allocating a portion of the $1600 to \(A_3\) (which will, of course, be borne by the victim \(A_0\)).

\(^{63}\) The purpose of this example is merely to illustrate the operation of the rules. It is a non-equilibrium example because, as we show in Section III, rational actors would not choose to be negligent under unitary share rules.

\(^{64}\) In fact, when we presented an earlier draft of this Article at faculty workshops, the participants suggested even more methods of apportionment. We agree that our taxonomy does not capture the full extent of plausible rules, but believe that it provides a useful way to classify the predominant
FIGURE I: APPORTIONMENT UNDER NEGLIGENCE

Full Liability Rules

<table>
<thead>
<tr>
<th>Unitary Share</th>
<th>Proportional Share</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed Share</td>
</tr>
<tr>
<td>Rule 1</td>
<td>A₀ = 0</td>
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<td>A₁ = 1250</td>
<td>A₁ = 937</td>
</tr>
<tr>
<td>A₂ = 1250</td>
<td>A₂ = 1563</td>
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</tr>
<tr>
<td>Rule 4</td>
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<tr>
<td>A₁ = 833</td>
<td>A₁ = 750</td>
</tr>
<tr>
<td>A₂ = 833</td>
<td>A₂ = 1250</td>
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</table>

Partial Liability Rules

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<th>Proportional Share</th>
</tr>
</thead>
<tbody>
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<td>Fixed Share</td>
</tr>
<tr>
<td>Rule 5</td>
<td>A₀ = 900</td>
</tr>
<tr>
<td>A₁ = 800</td>
<td>A₁ = 600</td>
</tr>
<tr>
<td>A₂ = 800</td>
<td>A₂ = 1000</td>
</tr>
<tr>
<td>Rule 7</td>
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<td>A₁ = 533</td>
<td>A₁ = 480</td>
</tr>
<tr>
<td>A₂ = 533</td>
<td>A₂ = 800</td>
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<table>
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<tr>
<td>Rule 8</td>
<td>A₀ = 1433</td>
</tr>
<tr>
<td>A₁ = 533</td>
<td>A₁ = 480</td>
</tr>
<tr>
<td>A₂ = 533</td>
<td>A₂ = 800</td>
</tr>
</tbody>
</table>

III. APPORTIONMENT UNDER NEGLIGENCE

In this Section, we evaluate the efficiency properties under negligence of the eight classes of rules of apportionment defined above. Throughout this Section, we assume that the standards of care are set at the socially optimal level. We show that the four classes of unitary share rules are always efficient, but that the other four classes of rules generally are not. Because, as we have explained, rules of joint and several liability are unitary share rules, we then discuss the conditions under which the common law holds actors jointly and severally liable, and provide an economic reinterpretation of the legal categories that give rise to such liability. Furthermore, we

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common law approaches. The proofs in the Appendix cover a wider (though not exhaustive) class of rules than that described in the text.

65. See supra text accompanying note 12.
explain why the imposition of joint and several liability does not necessarily give rise to a trade-off between fairness and efficiency.

A. Efficiency of the Various Rules

In their study of joint tortfeasors, Landes and Posner considered the efficiency properties of a single rule of apportionment, which under our taxonomy would be classified as a full liability, unitary, fixed share rule (Rule 1). They proved that such a rule produces the efficient outcome, but did not seek to ascertain the boundaries of the class of efficient rules or inquire whether the legal system contains rules that are not efficient.

We show that of the eight rules described in Section II, the four unitary share rules (Rules 1, 2, 5, and 6) lead to the maximization of social welfare under negligence, provided that the standards of care for each of the actors are set at the socially optimal level. Thus, every full liability, unitary share rule (Rules 1 and 2) and every partial liability, unitary share rule (Rules 5 and 6) is efficient. Under such rules, the efficient outcome is attained regardless either of the benefit and loss functions or of the number of actors contributing to the damage. The class of rules examined by Landes and Posner is a special case of this general category of efficient rules.

We also show that not all of the apportionment rules currently employed by the courts are efficient. In particular, no partial liability, fractional share rule (Rules 7 and 8) ever induces efficient behavior. Moreover, the two full liability, fractional share rules (Rules 3 and 4) are inefficient for certain benefit and loss functions and number of actors contributing to the damage.

In the Appendix, we prove that for the efficient outcome to occur, the tortfeasors must, in the aggregate, be responsible at least for the increased loss resulting from departures from the standard of care, including the loss attributable to non-negligent actors. We also prove in the Appendix that as long as rules are unitary rather than fractional, it does not matter, from an efficiency perspective, whether they are full liability rules rather than partial liability rules, or fixed share rules rather than proportional share rules.

The intuition behind the result that unitary share rules will induce efficient behavior is straightforward. Consider first the case of a partial liability, unitary share rule. Under this rule, the negligent actors will be liable for the full damage caused minus the damage that results when all actors are non-negligent.

66. See supra text accompanying notes 43-45, 51.
67. For each actor, the social optimum is the level that produces the maximization of the social objective function. See supra text accompanying notes 22-23. We deal in Section VI with standards of care that are not set at the optimal level.
If all but one of the actors are non-negligent, it would not be rational for the remaining actor to be negligent. If this actor were contemplating dumping more than the standard of care, she would face liability for the full increase in the resulting damage. If the standard of care is set at the social optimum, the increased benefits that this actor would obtain through negligent conduct would be less than the increase in the damage for which she would be liable. Thus, assuming that all but one of the actors are non-negligent, the remaining actor will be non-negligent as well.

Now consider whether it would be rational for more than one actor to be negligent. A unitary share rule will assign to the negligent actors, in the aggregate, the full increase in the ensuing damage. If negligent action on the part of these actors were preferable to non-negligent action for each of them, then the total social welfare would exceed that attainable when all actors meet the standard of care, which is not possible if the standard of care is set at the social optimum.

Consider first the group of non-negligent actors; none of them either gains or loses from a change in the behavior of an actor who chooses to become negligent, because they bear none of the loss and receive none of the resulting benefits. The negligent actors, on the other hand, would capture the full additional benefits of dumping more than the standard of care, but would pay for the full additional damage. Since social welfare is maximized when all the actors meet the standard of care, the increase in damages must exceed the increase in benefits.

Thus, it would not be rational for all these actors to be negligent. However the additional damage was allocated among them, at least some of them would have to pay more than the additional benefit that they obtained by acting negligently, since the aggregate increase in benefit is less than the aggregate increase in damage. Any actors who had to pay more than their increased benefits would opt to be non-negligent. But once those actors chose to be non-negligent, the apportionment rule would allocate to other actors damages exceeding their increased benefit, since regardless of how many actors are negligent, the increase in aggregate damage caused by that negligence would continue to be greater than the increase in aggregate benefit. It is therefore not rational for more than one actor to be negligent when a partial liability unitary share rule is in force.

We have already explained that it is not rational for a single actor to be negligent either. Thus, under a partial liability, unitary share rule, all actors will be non-negligent, and the efficient result will be attained.

68. It would never be economically rational for an actor to dump less than the standard of care, since the actor would bear no portion of the damage even if she dumped an amount equal to the standard of care. Thus, the actor would merely forego the benefit of engaging in the economic activity that produces the wastes.

69. See supra text accompanying notes 22-23.
This discussion shows that any partial liability, unitary share rule is efficient, whatever additional rules are employed to apportion the damage among negligent parties. Thus, both the partial liability, unitary, fixed share rule (Rule 5) and the partial liability, unitary, proportional share rule (Rule 6) lead to the efficient outcome.

It is now easy to show that any full liability, unitary share rule (Rules 1 and 2) is efficient as well. Indeed, if the liability for the increase in damage caused by negligence, which is imposed by a partial liability rule, deters all actors from being negligent, then surely liability for the increase in damage caused by negligence plus liability for the damage that results when all actors are non-negligent, which is imposed by a full liability rule, would also deter all actors from being negligent. Similarly, a full liability, unitary share rule will be efficient, whatever additional rules are employed to apportion the damage among negligent parties. Thus, both the full liability, unitary, fixed share rule (Rule 1) and the full liability, unitary, proportional share rule (Rule 2) lead to the efficient outcome.

In contrast to unitary share rules, fractional share rules are not generally efficient. Consider first the two partial liability, fractional share rules (Rules 7 and 8). In the Appendix, we prove that these rules always lead to an inefficient result. The intuition behind the result can be simply stated for a fixed share rule (Rule 7). Assume that all the actors are non-negligent, and that one actor contemplates dumping more than the standard of care. Because the actor would pay only a fraction of the increase in the damage, there will be a divergence between the actor’s private objective function and the social objective function—she will be responsible for less than the damage that she imposes on society. Thus, the actor will have an economic incentive to dump more than the standard of care.

The two remaining rules are the two full liability, fractional share rules (Rules 3 and 4). As we show in the Appendix, whether these rules will induce the efficient result depends on the benefit and damage functions and on the number of actors. The intuition proceeds as follows for a full liability, fractional, fixed share rule (Rule 3). In the discussion of partial liability, fractional share rules, we noted that the private cost of dumping more than the standard of care is less than the social cost; thus, negligent action will result. But for full liability, fractional share rules, a negligent actor would bear not only a fraction of the increased damage that results from her negligence, but also a fraction of the damage that results when all actors are non-negligent. Whether the total private cost that an actor contemplating negligent action must face is greater than the social cost

70. It follows that, without uncertainty, a rule that imposed even higher damages upon the tortfeasors would also be efficient. But because punitive damages are generally unavailable, see supra note 28, it is important to determine which rules will provide efficient incentives in the absence of such damages. As we show, unitary share rules provide the minimum incentive necessary to induce efficient behavior.
depends on the damage that results at the social optimum, which in turn depends on the benefit and damage functions and on the number of actors.

The efficiency properties of the eight rules are summarized in Figure II. All four unitary rules are efficient; the two full liability, fractional share rules are sometimes inefficient; and the two partial liability, fractional share rules are always inefficient.

While we recognize that considerations other than economic efficiency are cognizable in assessing the relative merits of the various rules, our conclusion is certainly relevant to that assessment. Proponents of inefficient apportionment rules should have the burden of explaining, on non-efficiency grounds, why such rules are preferable to efficient rules. Moreover, our conclusion that the choice of proportional share rules over fixed share rules is neither a necessary nor a sufficient condition for efficiency sheds light on the debate between these two rules, a debate that has been informed, at least in part, by notions of efficiency.

FIGURE II: EFFICIENCY OF THE APPORTIONMENT RULES UNDER NEGLIGENCE

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<tr>
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<th>Proportional Share</th>
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<tr>
<td>Full Liability Rules</td>
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<td>Rule 2</td>
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<td>Efficient</td>
</tr>
<tr>
<td>Fractional Share</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rule 3</td>
<td>Rule 4</td>
</tr>
<tr>
<td></td>
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<td>Sometimes</td>
</tr>
<tr>
<td></td>
<td>Inefficient</td>
<td>Inefficient</td>
</tr>
<tr>
<td><strong>Partial Liability Rules</strong></td>
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<tr>
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</table>

71. See supra text accompanying notes 51-56.
72. See United States v. Reliable Transfer, Inc., 421 U.S. 397, 405 n.11 (1975) ("A rule that divides damages by degree of fault would seem better designed to induce care than the rule of equally divided damages, because it imposes the strongest deterrent upon the wrongful behavior that is most likely to harm others.").
B. Reconsidering Joint and Several Liability

Our conclusion that only unitary share rules are efficient has important implications for the question of when joint and several liability is appropriate. As we have discussed, a rule of joint and several liability is a unitary share rule, since negligent actors bear damages attributable to non-negligent actors; thus, joint and several liability induces efficient outcomes.\(^7\) In turn, a rule of non-joint (several only) liability is a fractional share rule and, in general, is not efficient.\(^4\)

We do not mean to suggest, however, that whenever an individual is harmed by several actors, efficiency will be attained only if each negligent actor is held jointly and severally liable for the full damage. For example, if an individual has one car damaged by a negligent actor and a different car damaged by a non-negligent actor, the negligent actor need not be held jointly and severally liable for the damage to both cars. We therefore consider when joint and several liability (or a unitary share rule) is necessary to produce an efficient outcome. We also provide an economic reinterpretation of the concepts “distinct,” “divisible,” and “indivisible” harms, 73. One student commentator recently suggested that joint and several liability leads to over-deterrence. See Note, supra note 17, at 648–49. She notes that “[b]y eliminating joint and several liability, legislatures have eliminated an element of an actor’s expected accident costs. An actor’s expected accident costs no longer depend upon the probability that another actor who contributes to the victim’s injury will be insolvent or immune from suit.” Id. at 648. Therefore, she argues, “[j]oint and several liability sometimes forces an actor to pay for costs that do not result from his activity. These forced costs lead to a misallocation of resources because actors spend too much on safety.” Id. at 649; see id. (“If the tort system holds an actor liable for costs greater than those associated with his activity, he will not engage in the most efficient level of activity—he will be too safe.”). But even where there is the possibility of insolvency, which we have not explicitly addressed in this Article, see supra note 29, joint and several liability produces no over-deterrence under negligence, because an actor can avoid all liability by meeting the standard of care. Moreover, as we show below, see infra text accompanying notes 94–99, if the rule of liability is strict liability rather than negligence, there will be under-deterrence even if joint tortfeasors are held jointly and severally liable, because an actor thinking about departing from the social optimum does not see the full social cost of such a departure.

The student commentator also suggests that the absence of joint and several liability would be desirable from an efficiency perspective. See Note, supra note 17, at 649 (“When states limit joint and several liability, however, actors’ liability corresponds more closely to activity-related costs. Thus, although limiting joint and several liability reduces the tort law’s deterrent effect, it does so in a carefully tailored manner, consistent with the efficiency limitations of deterrence.”). As we have indicated in the preceding section, however, rules of non-joint liability always produce under-deterrence because they do not force an actor contemplating a violation of the standard of care to “see” the full social cost of such a violation. See supra text accompanying notes 69–71.

For other commentaries that have supported the abolition of joint and several liability, see, e.g., Pressler & Schieffer, supra note 17, at 682–84; Note, Compensation, Fairness, and the Costs of Accidents—Should Pennsylvania’s Legislature Modify or Abrogate the Rule of Joint and Several Liability Among Concurrently Negligent Tortfeasors?, 91 DICK. L. REV. 947, 974–76 (1987).

A recent article has suggested that economic theory provides no basis for determining how to allocate damages among multiple tortfeasors. See Wright, supra note 5, at 1169–79. However, under the four alternatives considered in that piece, the negligent defendants are liable, in the aggregate, for at least the full amount of the plaintiff’s damage. Id. at 1172. Thus, this commentator does not consider situations, such as those present under partial liability rules and fractional share rules, where the plaintiff recovers less than her full damage.

\(^{74}\) A fractional share rule can be efficient only in the case of distinct damages, which are defined infra text accompanying note 82.
which are used in the Restatement and in the common law to determine when joint and several liability will attach.

The Restatement defines “distinct” harms as harms “which, by their nature, are more capable of apportionment.” The Restatement further classifies harms that are not distinct as either divisible or indivisible. It states: “There are other kinds of harm which, while not so clearly marked out as severable into distinct parts, are still capable of division upon a reasonable and rational basis, and of fair apportionment among the causes responsible.” Three categories are therefore relevant: distinct, non-distinct but divisible, and non-distinct and indivisible. Under the Restatement's approach, joint and several liability attaches only in the case of non-distinct and indivisible harms.

In defining the three categories, the Restatement contemplates that a harm can be apportionable in four different senses. A distinct harm is one that is “more” capable of apportionment; a non-distinct but divisible harm is one that is somewhat less capable of apportionment; and a non-distinct and indivisible harm is presumably even less capable of apportionment. But even if a harm is non-distinct and indivisible, the defendant held jointly and severally liable may have a right to contribution, with the amount of such contribution determined by reference to the “equitable shares” of each of the defendants. This division into equitable shares is, then, another way of apportioning the damage. Not surprisingly, great confusion surrounds the question of when tortfeasors should be held jointly and severally liable.

We believe that the terms “distinct,” “divisible,” and “indivisible” should be reinterpreted in light of the efficiency properties of apportionment rules. Joint and several liability should attach, at least presumptively, when fractional rules would not guarantee an efficient outcome. As we have indicated, we recognize that tort law can serve goals other than economic efficiency. However, when an inefficient rule is chosen over an efficient one, some non-efficiency justification must be provided. It is un-

75. RESTATEMENT, supra note 10, § 433A comment b.
76. Id. § 433A comment d.
77. See id. §§ 875, 881.
78. See id. § 886A.
79. We offer one example of the confusion. Under CERCLA, courts have held that where the harm caused by the disposal of hazardous wastes by several liable actors is indivisible, any liable actor can be held jointly and severally liable, but such an actor will be able to seek contribution. The relevant equitable shares will be determined by reference to such factors as the amount of wastes disposed by each actor and the toxicity of such wastes. See, e.g., Colorado v. ASARCO, Inc., 608 F. Supp. 1484 (D. Colo. 1985); United States v. Chem-Dyne Corp., 572 F. Supp. 802, 810–11 (S.D. Ohio 1983). But in what appears to be a directly analogous situation, the Restatement adopts the opposite position. See infra text accompanying note 88 (discussing RESTATEMENT, supra note 10, § 433A comment d). In the example presented by the Restatement, unlike under CERCLA, the estimate of the quantities of pollution generated is used to defeat the plaintiff’s claim of joint and several liability, rather than to calculate shares of contribution following the imposition of such liability.
80. Discussion of these issues is beyond the scope of this Article.
likely that the muddled approach of the Restatement, with different consequences attaching to the ease with which a harm is "capable of apportionment," evinces a concern for such goals.

From an efficiency perspective, there is no need to have three different categories of harms, since there are only two possible legal consequences: imposition or rejection of joint and several liability. Because there is no legal significance to the distinction between distinct harms on the one hand and non-distinct but divisible harms on the other, we do not think that it is helpful to separate harms along these lines. To the contrary, the multiplicity of unnecessary legal categories is quite likely to breed confusion.

Thus, we think that harms should be classified as either distinct or non-distinct; that joint and several liability should attach only to non-distinct harms; and that the classification should aim to isolate a category of harms for which joint and several liability is not necessary to induce efficient outcomes.

Under these principles, a harm is distinct if the damage caused by one actor is independent of the harm caused by other actors. In contrast, where the damage caused by one actor increases the damage caused by another actor, the harm is non-distinct. In our hazardous waste example, the expected damage of the dumping is the square of the total amount dumped. Assume that there are two actors. If the first actor dumps five units and the second does not dump anything, then the harm is $25. But if the second actor were also to dump five units, the total harm would not be $50 (twice $25) but $100 ($10 squared). Thus, the second actor can be said to have caused a damage of $75 by dumping, even though this damage would have been only $25 if the first actor had not dumped as well.

At least in part, the Restatement appears to embody a similar definition. Describing an example of distinct harms, it states:

If two defendants independently shoot the plaintiff at the same time,
and one wounds him in the arm and the other in the leg, the ultimate result may be a badly damaged plaintiff in the hospital, but it is still possible, as a logical, reasonable, and practical matter, to regard the two wounds as separate injuries, and as distinct wrongs. The mere coincidence in time does not make the two wounds a single harm, or the conduct of the two defendants one tort.\footnote{84}{RESTATEMENT, supra note 10, § 433A comment b.}

Up to this point, the example appears to be consistent with our definition of distinct harms as ones for which the damage function is separable. The damage caused by the wound in the arm would have been the same even if there had been no wound in the leg; similarly, the damage caused by the wound in the leg would have been the same even if there had been no wound in the arm.

The Restatement, however, goes on to say:

There may be difficulty in the apportionment of some elements of damages, such as the pain and suffering resulting from the two wounds, or the medical expenses, but this does not mean that one defendant must be liable for the distinct harm inflicted by the other. It is possible to make a rough estimate which will fairly apportion such subsidiary elements of damages.\footnote{85}{Id.}

While this statement is ambiguous, it can be read to suggest that the two wounds would be considered distinct even if the presence of one increased the pain and suffering or medical expenses that resulted from the other.\footnote{86}{It may be that the Restatement refers to these as “subsidiary” damages because they involve comparatively small amounts. Certainly with respect to pain and suffering, that is no longer true.} The second half of the example therefore muddles the definition and compromises an efficient outcome.\footnote{87}{Also, to the extent that a “rough estimate” has to be made, why is the harm distinct rather than non-distinct but divisible?}

The Restatement’s discussion of divisible harms (for which joint and several liability does not attach) also appears to be at odds with notions of economic efficiency. It uses joint pollution to illustrate divisible harms:

[W]here two or more factories independently pollute a stream, the interference with the plaintiff’s use of water may be treated as divisible in terms of degree, and may be apportioned among the owners of the factories, on the basis of evidence of the respective quantities of pollution discharged into the stream.\footnote{88}{RESTATEMENT, supra note 10, § 433A comment d.}

But it is quite unlikely that the damage function in the case of joint pollution would be linear, and therefore that the damage caused by one factory would be independent of whether another factory was also polluting. For
many pollutants, the damage function exhibits thresholds at which there are marked non-linearities. For example, as the concentration of dissolved oxygen in water decreases toward a threshold, there will be a pronounced increase in the damage caused, since the water will no longer be able to support fish.  

Thus, one polluter's discharge will affect the harm another causes, and the damage function will therefore not be separable. Proper attention to considerations of economic efficiency would lead to the classification of such harms as non-distinct and to the application of joint and several liability.

C. Unitary Share Rules and Causation

At first glance, it would appear that the requirement of causation as a predicate for tort liability argues for a fractional share rule over a unitary share rule, since negligent actors are not responsible for harms that are attributable to non-negligent actors. To the extent that considerations of fairness dictate attention to causation, rules of apportionment might appear to present a trade-off between efficiency and fairness, with efficiency considerations supporting unitary share rules and fairness considerations supporting fractional share rules.

In fact, the relationship between unitary share rules and causation is not quite so straightforward. The portion of the damage that the negligent parties cause can be seen as the increase in the damage those parties produce. Admittedly, under a unitary share rule, the negligent parties pay more than this amount. But under a fractional share rule, as we have shown, they pay less than the increase in the damage attributable to them. Indeed, it is precisely for this reason that fractional share rules are not generally efficient. In terms of fairness, then, fractional share rules, like unitary share rules, are problematic. There is no reason to prefer


91. See supra text accompanying notes 69–71. This point appears to be lost on some of the literature that advocates limitations on joint and several liability. For example, a student commentator notes that “[j]oint and several liability sometimes forces an actor to pay for costs that do not result from his activity. . . . When states limit joint and several liability, however, actors’ liability corresponds more closely to activity-related costs.” Note, supra note 17, at 649. She refers primarily to a solvent negligent actor's responsibility for damages attributable to a negligent actor who is insolvent, see id. at 648, but her argument applies with equal force to a solvent negligent actor's responsibility for damages attributable to non-negligent actors. See also Note, supra note 73, at 975–76.
charging tortfeasors less than the damage they cause to charging them more.

In particular cases, it may be that a fractional share rule slightly undercharges the negligent actors, while a unitary share rule overcharges them a great deal. For other benefit and damage functions, however, the converse will be true. Even benefit and damage functions for which fractional share rules do not significantly undercharge when the number of actors is small will undercharge more as the number of actors increases. Thus, it is not possible to make a general statement about whether causation and fairness considerations recommend the use of fractional share rules rather than unitary share rules. For this reason, the choice between unitary share and fractional share rules does not give rise to a general trade-off between fairness and efficiency.

IV. APPORTIONMENT UNDER STRICT LIABILITY

In this Section, we turn to the analysis of strict liability rules. We first show that none of the apportionment rules that courts currently use and commentators advocate are efficient. We then demonstrate the possibility of plausible apportionment rules that produce the efficient outcome under strict liability. To our knowledge, the courts have not used nor have commentators advocated such rules.

A. Existing Apportionment Rules

Under negligence, we have seen that four classes of rules are efficient if the standards of care are set optimally. Under strict liability, in contrast, none of the classes is efficient.

Under strict liability, the range of apportionment rules is much more constrained than under negligence. As we have indicated, of the eight types of rules that we considered in the context of negligence, only two are relevant under strict liability: fixed share rules and proportional share rules.

We prove in the Appendix that these apportionment rules are inefficient under strict liability. The intuition behind the result is simple for fixed share rules. Assume that all actors are dumping the optimal amount of waste and that one actor considers whether to dump more. This actor would bear not the full increase in the damage but only a portion of that increase, determined by his fixed share. This creates a divergence between the actor's private objective function and the social objective func-

92. See infra Appendix, Proposition 3.
93. See supra text accompanying notes 44, 49–58.
94. See S. SHAVELL, supra note 5, at 164–65.
This divergence will lead the actor to dump more than the social optimum. This intuition behind the reasons for the inefficiency of the existing apportionment rules under strict liability parallels the analysis that suggested the inefficiency of partial liability, fractional share rules under negligence. In both cases, an inefficient outcome results because an actor who contemplates dumping more than the socially optimal level does not bear the full increase in the damage that she imposes on society. In the case of the existing apportionment rules under strict liability, other actors must themselves bear a portion of the increase in the damage caused by the actor who dumps more than the social optimum. In the example above, such actors will be liable for a fixed share of a greater damage. For partial liability, fractional share rules under negligence, the increase in damage is instead borne by the victim.

The explanation for the efficiency of unitary share rules under negligence and the inefficiency of strict liability rules is that the former make an actor who departs from the optimal level of production of waste pay for the full increase in the damages she inflicts. Under strict liability, in contrast, such an actor pays for only a fraction of such damage.

This result does not depend on the discontinuity in the damage function that is presented by full liability rules under negligence, where an actor who dumps more than the socially optimal level must pay not only the

95. Landes and Posner use a short example to suggest that strict liability might be inefficient. See W. LANDES & R. POSNER, supra note 5, at 215; Landes & Posner, supra note 5, at 543. In their example, each of 10 polluters of a stream imposes costs of $1 million through his pollution. Five could avoid the pollution at a cost of $600,000 each, and the other five at a cost of $1.4 million each. Landes and Posner point out that under a negligence rule the first five would have incentives to avoid pollution, and society would therefore be wealthier by $2 million. They argue that under strict liability, however, the first five would not avoid pollution because “each will still have an expected liability of $500,000 (assuming either a no-contribution rule with each polluter equally likely to be sued for the remaining damage or a contribution rule in which damages are divided by the number of defendants).” Id.

They believe that if the five polluters were to eliminate their pollution they could still be held liable for a portion of the remaining damage of $5,000,000. It is certainly not clear why these polluters would be treated as joint tortfeasors, since they would not be polluting anything. If they were not treated as joint tortfeasors, then they would face the correct incentives and, in fact, would not pollute at all. Moreover, in this example, if a proportional share rule, rather than a fixed share rule, were used, strict liability would in fact produce the efficient result, since the five polluters could escape liability entirely.

What is peculiar about the Landes and Posner example is that under this example a strict liability rule can lead to the efficient allocation. Efficiency results in this example because under their formulation of the problem the optimal standard of care under negligence for the five polluters with lower abatement costs is zero pollution, and therefore strict liability yields the same results as negligence. Of course, in such cases, strict liability, like negligence, can lead to the efficient allocation, provided that an actor that produces no pollution is not held liable for any damage.

96. Full liability, fractional share rules are sometimes efficient because of the discontinuity that they exhibit in the damage function. An actor departing from the standard of care must pay not only her portion of the increased cost of that departure, but also the damage that is attributable to her non-negligent behavior. As we have shown, however, this discontinuity is not sufficient to guarantee efficiency. See supra Section III.A. (showing that full liability, fractional share rules are not always efficient).
increase in the damage caused by that departure from the optimum but also the damage that would have resulted in the absence of negligence. Indeed, partial liability rules, for which the damage function is not discontinuous, are efficient as well, because they allocate the full increase in the damage to the actor who departs from the socially optimal level.

Although rules of joint and several liability are efficient under negligence, they will not produce efficient results under strict liability. In fact, assuming that all parties are solvent and that transaction costs are zero, as we have assumed for negligence rules, the question of whether the harms are deemed distinct, divisible, or indivisible is of no economic consequence for strict liability rules. Under negligence, the characterization of the harm is important because it determines whether the negligent actors must pay for the damage attributable to non-negligent actors. But under strict liability, this characterization has no consequence because all of the actors violate the standard of care.

We do not present here a general argument for negligence over strict liability. Many reasons other than economic efficiency may underlie a preference for one rule over another. In particular, if the standard of care under negligence is not set at the social optimum, negligence rules will not, in general, be efficient. Also, the distributional consequences of both rules are markedly different because under negligence, the victim often bears larger losses than under strict liability. Thus, in the case of joint harms, strict liability rules must be justified by reference either to the practical difficulties involved in setting the standard of care optimally, or to non-efficiency goals that are sufficiently compelling to outweigh the efficiency benefits of negligence rules.

B. **Efficient Apportionment Rules**

While the existing apportionment rules are not efficient under strict liability, we propose apportionment rules that do induce the efficient outcome. All such rules take into account whether an actor is dumping more wastes than the social optimum. In the Appendix, we formally define a class of efficient rules. Here, we suggest a simple rule in this class and show why it is efficient. Under this rule, the full damage is divided per capita among the various actors when none dump more than the socially optimal level. If some actors dump more than this level, then those actors divide the full damage among themselves per capita, and the remaining actors pay nothing.

Such a rule is efficient. If all but one of the actors meet the socially optimal level, it would not be rational for the remaining actor to dump

97. See *supra* text accompanying notes 68-70.
98. See *supra* text accompanying note 50.
99. See *infra* Section VI.
more than the socially optimal amount. If she were to increase her output beyond the social optimum, she would bear the full increase in the ensuing damage. In addition, she would have to pay the full damage that occurs when all actors meet the socially optimal level, rather than the per capita share of that damage, which she would pay if she did not depart from this level. In Section III, we showed in connection with partial liability, unitary share rules, that even if the only cost of departing from the social optimum was the increase in the damage, it would not be rational for an actor to dump more than that optimal level, because the additional damage for which the actor would be responsible would be greater than the additional benefit derived by such actor. Thus, when there is also another cost to departing from the social optimum, as there is here, it follows a fortiori that such a departure would not be rational.

Similarly, it would not be rational for a group of actors to dump more than the socially optimal level. The aggregate cost to this group of doing so is the full increase in damage resulting from such a departure, plus the portion of the damage borne by the other actors when all the actors meet the socially optimal level. We showed in Section III that even if these actors had to pay only for the increase in damage, departure from the social optimum would not be rational. Such a departure would similarly not be rational when these actors must pay not only this amount but also an additional component of damages.100

Under these rules of apportionment, the allocation of the loss depends to a great extent on whether the actors meet the socially optimal level. These rules, therefore, replicate for strict liability the desirable economic features of negligence rules.101

These efficient apportionment rules under strict liability are no more cumbersome for courts to apply than are traditional negligence rules, since both sets of rules turn on the determination of the socially optimal level. We recognize, however, that both negligence rules and the strict liability rules that we propose—unlike traditional strict liability rules—require that, for each actor, courts determine the socially optimal level of dump-

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100. *See supra* text accompanying note 70. We prove in the Appendix that an efficient outcome would also occur if the full damage were divided proportionally among all of the actors when none dumped more than the socially optimal level, and, otherwise, proportionally among those who dump more than that level.

101. They do so by introducing what might be called "standards of apportionment" that serve the same function as standards of care under negligence. Because of this parallelism between standards of care and standards of apportionment, one could introduce classes of untraditional strict liability rules that correspond to the fractional share and unitary share rules defined under negligence. In strict liability, however, one must be careful to distinguish between rules that are unitary with respect to standards of care and rules that are unitary with respect to standards of apportionment. Every strict liability rule is unitary with respect to standards of care. *See supra* text accompanying notes 46–49. The efficient rules we propose are also unitary with respect to standards of apportionment. One can easily construct rules that are fractional with respect to standards of apportionment.

We do not pursue the analysis of unitary versus fractional share rules with respect to standards of apportionment because it would virtually duplicate that of Section III.
ing. If the costs of doing so are great, or if courts are unable to determine the social optimum,\textsuperscript{102} traditional strict liability rules might be preferred over both negligence rules and efficient strict liability rules.

The efficient apportionment rules under strict liability that we have defined permit the disaggregation of efficiency from distributional concerns. As we have noted, under strict liability rules, by definition, the victim gets compensation for her full harm.\textsuperscript{103} Under negligence rules, in contrast, full compensation will not be available in many instances, most notably where all the actors meet the standard of care.

Thus, under the current rules of apportionment, the choice between negligence and strict liability gives rise to a trade-off between efficiency and distributional goals. The decisionmaker (whether a court or a legislature) can achieve full compensation for the victim only at the cost of foregoing the efficiency benefits of negligence rules. In contrast, under the apportionment rules that we propose, the choice between negligence and strict liability need not present such a trade-off. To the extent that the distributional consequences of strict liability are desirable, they can be achieved without any loss in efficiency.

V. Rules of No Contribution

So far, we have analyzed joint and several liability (unitary share rules) in connection with a right to contribution.\textsuperscript{104} Here, we study rules of no contribution, which attracted the attention of the law-and-economics literature in the early 1980s. The properties of no-contribution rules shed light on the relative desirability of the contribution rules that we have discussed.

Landes and Posner make two claims about rules under which an actor who has been held jointly and severally liable cannot obtain contribution from other actors. First, they argue that rules of no contribution are efficient; and second, they argue that such rules produce incentives identical to those produced by rules of contribution.\textsuperscript{105} Their result holds under negligence, on which they focused, but does not hold under strict liability.

A no-contribution rule is equivalent to a fixed share rule in which an actor's share is her estimate of the probability that she will be the one to be held jointly and severally liable and therefore responsible for the full damage.\textsuperscript{106} Thus, the properties of no-contribution rules follow simply from our discussion of rules of contribution.

\textsuperscript{102} We address this problem in Section VI.

\textsuperscript{103} See supra text accompanying note 44.

\textsuperscript{104} Of course, in the absence of joint and several liability, there is no right to contribution, since such a right can be invoked only by an actor who has paid more than his equitable share of the damage. See Restatement, supra note 10, § 886A(2). Thus, for fractional rules, there can be no right to contribution.

\textsuperscript{105} See Landes & Posner, supra note 5, at 529-30.

\textsuperscript{106} Her expected damage is this probability multiplied by the cost of the full damage. The
Under negligence, we showed that apportionment rules are efficient if the sum of the shares of the negligent actors is equal to one, as it is for unitary share rules. We also showed that apportionment rules are not efficient if the sum of the shares is less than one, as it is for fractional share rules.

For a no-contribution rule to be efficient, however, the sum of probabilities that the actors attach to the risks of being held responsible for the full damage must be equal to at least one. If each actor’s estimate of his probability of being held liable were known to all the other actors, one would expect that the sum of probabilities would be equal to exactly one. If, in contrast, the sum of these probabilities is less than one, the no-contribution rule will be equivalent to a fractional share rule, and will therefore not generally be efficient.

Under strict liability, a rule of no contribution will generally not lead to an efficient allocation. Recall from our earlier discussion that there will be a divergence between the private objective function and the social objective function, unless an actor who contemplates dumping more than the socially optimal level bears the full increase in the damage caused by such a departure. Under no-contribution rules, this result will occur only if each actor estimates that the probability of being responsible for the full damage is one. Thus, if the probabilities are common knowledge and therefore can be expected to sum to one, no-contribution rules are not efficient under strict liability.

It also follows that, under strict liability, there is little reason to suspect that a rule of no-contribution will lead to the same behavior as a rule of apportionment. Such equivalence will prevail only if each actor faces the identical incentives under both regimes. But this will happen only if each actor’s share under the rule of apportionment is equal to her expected liability under the no-contribution rule. Such a coincidence seems unlikely, particularly under a proportional share rule, where an actor’s share depends not only on her own decision, but also on the decisions of all the other actors.

It bears noting that while no-contribution rules will not lead to the efficient result under strict liability, rules of contribution, if properly designed, can do so, as we have explained in Section IV.B. Thus, we present an argument for allowing actors held jointly and severally liable under strict liability rules to seek contribution from other responsible actors.

argument that no-contribution rules are efficient assumes that all actors are risk-neutral. See supra note 8. We assume throughout this discussion that a single defendant will pay the full judgment.

107. We have shown that the result will be efficient if the shares add to one. If they add to more than one, the same result will hold; with no uncertainty, there is no danger of over-deterrence because the actors can avoid all liability by meeting the standard of care.

108. Landes and Posner assumed that the sum of the probabilities would equal one. See Landes & Posner, supra note 5, at 522.
VI. DEPARTURES FROM THE SOCIAL OPTIMUM IN THE DETERMINATION OF THE STANDARD OF CARE

To this point, we have assumed that courts can properly define the social optimum for purposes of setting the standard of care under negligence and for applying the efficient strict liability rules that we propose.\(^{109}\) We have made this assumption for several reasons.

First, the traditional law-and-economics models of tort law for single injurers,\(^{110}\) as well as the models for joint tortfeasors, have made the same assumption.\(^{111}\) Thus, our conclusion that, under a similar assumption, certain common law rules are not efficient contradicts the central tenet of the positive economic theory of tort law.\(^{112}\)

Second, under certain judicial formulations of the standard of care, such as the Hand formula,\(^{113}\) courts purport to set this standard at the social optimum. Regardless of what courts actually do in practice, it is relevant to study the effects of the rules that the courts say they are applying.

Third, when analyzing a legal problem, it is important to separate it into its components. By looking initially at the effects of apportionment rules when standards of care are set at the socially optimal levels, we are able to distinguish, on the one hand, the effects that result from the "jointness" of the problem, and on the other, the effects that stem from "error" in the determination of the standard of care.

Therefore, an analysis of rules of apportionment does well to begin with the assumption that the standard of care is set at the socially optimal level. Any policy recommendation, however, must confront the reality that the standards of care governing conduct will often not be optimal.

At the outset, we must distinguish two principal ways in which the standard of care might deviate from the socially optimal level. First, courts might attempt to set the standard of care at the optimal level but might make errors in determining the social optimum.\(^{114}\) Such errors might occur, for example, because courts have difficulty in ascertaining the exact benefits that dumpers receive from engaging in the activity that leads to the generation of wastes. Because the optimal standard is that which maximizes the sum of the benefits derived by the various dumpers minus the

\(^{109}\) For traditional strict liability rules, of course, there is no need to determine the social optimum.

\(^{110}\) See, e.g., Brown, supra note 30.

\(^{111}\) See Easterbrook, Landes & Posner, supra note 5; Landes & Posner, supra note 5; Polinsky & Shavell, supra note 5.

\(^{112}\) The positive theory states that common law tort rules are efficient. For a comprehensive statement of the theory, see Landes & Posner, The Positive Economic Theory of Tort Law, 15 GA. L. Rev. 851 (1981).

\(^{113}\) See United States v. Carroll Towing Co., 159 F.2d 169, 173 (2d Cir. 1947) (L. Hand, J.).

\(^{114}\) Similarly, courts might err in ascertaining the levels of care adopted by the actors. This problem is conceptually analogous, though not identical, to that posed when courts make errors in determining the socially optimal level.
social loss, an incorrect estimation of the benefits will lead to an incorrect standard of care.

When courts make such errors, the standard of care must be described by means of a probability distribution, rather than by a single number. We assume that the errors that a court makes in determining the social optimum are as likely to result in a standard that is too stringent as in one that is too lenient; we envision a symmetric distribution with a mean equal to the standard of care. For ease of exposition, our discussion proceeds by reference to a normal distribution.

We refer to the problem in which courts attempt to set the socially optimal standard of care but make errors in doing so as involving an optimal but uncertain standard. The standard can be said to be optimal in that its expected value is the socially optimal level. The standard is uncertain, however, in that an actor who dumps at the socially optimal level might nonetheless be found to violate the standard of care because, in a given case, the court might set this standard at a level more stringent than the socially optimal level. More generally, regardless of the amount that she dumps, the dumper will not be able to determine precisely whether she will be found to have violated the standard of care.

The second type of departure from the social optimum concerns a standard of care that is set at a level different from the social optimum, either too stringently or too leniently, but which does not exhibit uncertainty. Thus, a dumper will be able to determine precisely whether her conduct will constitute a violation of the standard. We refer to this problem as involving a non-optimal but certain standard of care.

A. Optimal Standards With Uncertainty

For single actors, the literature has extensively examined full liability rules and shown that models inducing the efficient result in the absence of uncertainty lead to inefficient results under uncertainty. In particular, uncertainty will produce over-deterrence if the standard deviation of the probability distribution is small and under-deterrence if it is large.

115. It is true, however, that the standard announced in a particular case may well not be optimal.

116. We do not deal in this discussion with standards that are non-optimal and uncertain.


In each of these works, uncertainty is modelled as risk. That is, the agent knows the probability distribution from which the actual standard of care is drawn (or, equivalently, knows the probability that she will be found non-negligent given that she has adopted a specific level of care). The degree of uncertainty is then measured by the dispersion in the probability distribution. Our discussion makes a similar assumption.
These models can easily be extended to show that partial liability rules unambiguously produce under-deterrence.\footnote{118}

In the Appendix, we extend these models to multiple actors and study how four of the apportionment rules perform under uncertainty.\footnote{119} As for single actors, two opposite incentives are present under full liability, unitary share rules. First, under uncertainty, if the actor dumps more than the optimal amount, it does not necessarily follow that she will be negligent, because there is some chance that the court will set the standard of care at an even less stringent level. This effect gives her an incentive to take too little care.

But a countervailing effect is also present. In the absence of uncertainty, an actor would not take care beyond the social optimum because she could avoid all liability simply by dumping the socially optimal amount. With uncertainty, however, there is the possibility that the court will announce a standard that is too stringent. Therefore, increasing one's level of care beyond the socially optimal level reduces the probability of being held liable.

This effect gives the actor an incentive to take too much care under full liability rules. If she is found negligent, she is responsible not only for the losses that result from her departure from the standard of care, but also for the losses that would have resulted even without negligence.

In contrast, this incentive toward over-deterrence is absent under partial liability rules. Under such rules, an actor is liable only for the damage that results from her departure from the standard of care. Thus, there will never be an incentive for such an actor to take more care than the social optimum because, by definition, the decrease in benefits will be greater than the increase in resulting damage, and even under a unitary share rule the actor would not be liable for more than this latter amount. Because the incentive to undercomply is present but the incentive to overcomply is not, partial liability rules always produce under-deterrence.

It follows that partial liability, unitary share rules are always preferable to partial liability, fractional share rules because the fractional element of the rule simply exacerbates the incentive to undercomply. Thus, partial liability, fractional share rules are never preferred.

The relative desirability of the three remaining classes of rules depends on the tightness of the probability distribution that describes the uncertainty surrounding the standard of care. While we have not attempted a systematic analysis of the problem, we have the following intuitions. As in the single-actor case, for "tight" distributions (small standard deviation)

\footnote{118. Consider, for example, the model of Craswell and Calfee. The incentives for overcompliance disappear under partial liability rules, leaving only incentives for undercompliance. See Craswel & Calfee, supra note 117, at 282.}

\footnote{119. In this discussion, we do not look at the difference between fixed share and proportional share rules.
the incentive to overcomply under full liability, unitary share rules is strong. This incentive, however, can be tempered in two ways: by making the rule a partial liability rule or by making it a fractional share rule. As we have stated, partial liability rules would eliminate the incentive to overcomply and would create an incentive to undercomply. Thus, when the distribution is very tight and full liability rules would produce too great an incentive to overcomply, it will be preferable to have undercompliance through a partial liability, unitary share rule.

Conversely, for distributions that are very disperse (large standard deviation), the incentive to undercomply is greater than the incentive to overcomply for all apportionment rules. Then, full liability, unitary share rules are best because they minimize the amount of undercompliance. In some intermediate range, full liability, fractional share rules are preferable because they weaken but do not eliminate the incentive to overcomply. The optimal rules derived by this analysis are shown in Figure III.

**FIGURE III: PREFERRED APPORTIONMENT RULES UNDER UNCERTAINTY**

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<th>Full Liability</th>
<th>Partial Liability</th>
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<td><strong>Unitary Share</strong></td>
<td>Disperse Distribution</td>
<td>Tight Distribution</td>
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<td>(high standard deviation)</td>
<td>(low standard deviation)</td>
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<tr>
<td><strong>Fractional Share</strong></td>
<td>Intermediate Distribution</td>
<td>Never Preferred</td>
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<td>(intermediate standard deviation)</td>
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**B. Non-Optimal Standards Without Uncertainty**

We consider first the case of a single actor. There are two relevant possibilities: either the standard of care is more lenient than the socially optimal level (permitting the dumping of more than the socially optimal amount of waste) or it is more stringent than that level (permitting the dumping of less than the socially optimal amount of waste).

Suppose first that the standard of care is too lenient. The actor will then dump the amount permitted by the standard of care, and therefore will dump more than the socially optimal amount. Indeed, the actor would capture the increased benefits of dumping more than this amount but would not be liable for any additional damages, as she could avoid all
liability by meeting the standard of care. This result holds for both full liability and partial liability rules, as in both cases excess dumping gives rise to additional benefits but no additional costs. Under both rules, the actor will dump the amount permitted by the standard of care.\textsuperscript{120}

When the standard of care is set too stringently, the analysis is somewhat more complex. If the actor meets the standard of care, therefore dumping less than the socially optimal level, the benefits that she foregoes by not dumping at this latter level exceed the corresponding increase in damages. Under a partial liability rule, an actor dumping at the socially optimal level would be liable only for these additional damages. Thus, it would be rational for the actor to violate the standard of care and dump the socially optimal amount.\textsuperscript{121}

Under full liability rules, in contrast, an actor dumping the socially optimal amount would be liable not only for the additional damages caused by dumping the socially optimal level rather than the amount prescribed by the standard of care, but also for the damages that result when the standard of care is met. The actor will meet the standard of care, thereby dumping less than the socially optimal amount, when the standard of care is not too much more stringent than the socially optimal amount. However, when the standard of care is far more stringent than the socially optimal amount, the actor will dump the socially optimal amount, thereby violating the standard of care.\textsuperscript{122}

Our problem, of course, concerns multiple actors, rather than single actors. To analyze it completely, one needs to know for each actor whether the standard has been set too stringently or leniently.\textsuperscript{123} While a comprehensive study of the issue is beyond the scope of this Article,\textsuperscript{124} we can draw some conclusions from a simplified problem in which there are two actors who have identical benefit functions (and therefore identical optimal levels of care), and who face the same standard of care.

If the standard of care is set more leniently than the socially optimal level, each actor will choose a level of care equal to the standard of care for every unitary share rule, whether full liability or partial liability. No actor will dump less than the standard of care, because she can obtain additional benefits at no cost by dumping up to the level permitted by the standard of care. Nor will any actor dump more than the standard of care. If all actors are dumping at the standard of care level, an actor thinking about dumping more than the standard of care would be liable at least for

\textsuperscript{120} The concepts of unitary and fractional share have no meaning for single tortfeasors, as the question of who pays for the damages caused by non-negligent parties does not arise.
\textsuperscript{121} See Kahan, supra note 30.
\textsuperscript{122} These results are summarized in S. Shavell, supra note 5, at 83.
\textsuperscript{123} Unless each actor faces identical, linear benefit functions, a stringent standard of care for one actor does not offset a lenient standard of care for another, because the benefits foregone by the former actor are not completely recouped by the extra benefits garnered by the latter actor.
\textsuperscript{124} We hope, however, to return to this question in the future.
the full increase in the damages, which, whether a partial liability rule or a full liability rule is used, will be more than the resulting benefits.\footnote{These damages would accrue under a partial liability rule. Under a full liability rule, the actor would also be liable for the damages that result when all actors are non-negligent.}

Moreover, there can be no equilibrium in which some actors dump more than the standard of care, when this standard is too lenient. As in the case of standards of care set optimally, such actors will be liable, in the aggregate, for damages greater than the ensuing benefits. Regardless of how the additional damage caused by the violation of the standard of care is apportioned among them, at least some actors would have to pay more than the additional benefit that they obtained through the violation.\footnote{See supra text accompanying notes 69–70.} Therefore, under both full liability, unitary share rules, and partial liability, unitary share rules, all actors will dump the amount prescribed by the standard of care when this standard is too lenient.

As is the case with single tortfeasors, the analysis is less straightforward when the standard of care is set too stringently. We first deal with partial liability, unitary share rules. Under a partial liability rule, a single tortfeasor would violate the standard of care and dump the socially optimal level. Similarly, there is no equilibrium in which any two actors would meet the standard of care because the benefits that one actor can gain by departing from the standard of care, given that the other adheres to the standard of care, are greater than the full increase in damages. Unlike the case of a single actor, however, for two actors there is no equilibrium in which the socially optimal level is met. An actor will dump at the socially optimal level only if she is liable for the full increase in damages caused by dumping more than the socially optimal level. But under a partial liability, unitary, fixed share rule, if one actor is dumping at the socially optimal level, the other actor would bear only half of the increase in damages that would result if she dumped more than the socially optimal level.\footnote{This effect is also present under proportional share rules.} Thus, the actor will dump more than the social optimum.

There is another candidate for an equilibrium—the equilibrium under strict liability. Our analysis of strict liability explains that if both actors are going to be negligent, they will each dump the amount that they would have dumped under a traditional strict liability rule (we shall refer to this level as the strict liability level).\footnote{We show in the Appendix that the total amount dumped under a strict liability rule is more than the socially optimal aggregate level. Thus, when the actors are identical, each will dump more than her socially optimal level.} Once both actors have decided to be negligent, the presence of the standard of care plays no role in their decisions.

The strict liability level will be an equilibrium if the standard of care is sufficiently more stringent than the socially optimal level. Then, if one actor is dumping at the strict liability level, the other actor would not be
better off by dumping at the level prescribed by the standard of care. Indeed, the positive net benefits obtained by moving from the standard of care to the social optimum will be greater than the negative net benefits that attach to a move from the socially optimal level to the strict liability level. Thus, under this condition, there will be an equilibrium in which both actors dump at the strict liability level.

In contrast, if the standard of care is less stringent (but still more stringent than the socially optimal level), one actor will prefer to dump at the level prescribed by the standard of care, given that the other actor is dumping at the strict liability level. Because we have shown that there is no equilibrium in which both actors meet the standard of care, it follows that, in such cases, there is no equilibrium in pure strategies. A full example showing these effects is presented in the Appendix.

The results are different for full liability, unitary share rules. Under such rules, an actor departing from the standard of care must bear not only the increased damages produced by this departure, but also the damages that result when both actors are non-negligent. Thus, for standards of care sufficiently close to the social optimum, there will be an equilibrium in which both actors meet the standard of care. For standards of care that are sufficiently more stringent than the social optimum, there will an equilibrium in which both actors dump at the strict liability level. The positive net benefits obtained by moving from the standard of care to the social optimum will be sufficient to counteract not only the negative net benefits that attach to a move from the socially optimal level to the strict liability level, but also the damages that result when both actors are non-negligent.

We show in the Appendix that there is an intermediate range in which the standard of care is neither sufficiently close to the socially optimal level for an equilibrium to exist in which each actor dumps the level prescribed by the standard of care, nor sufficiently far from the socially optimal level for each actor to dump at the strict liability level. In this range, there is no equilibrium in pure strategies.

The levels of care taken under full liability, unitary share rules and under partial liability, unitary share rules are set out in Figure IV for different standards of care. We denote the standard of care as $c$; the optimal level of care as $x^*$; the strict liability level of care as $x_s$; $a_1$, $a_2$, and $a_3$, are levels of care such that $0 < a_1 < a_2 < a_3 < x^*$. Figure IV shows that, when the standard of care is set at a non-optimal level, partial liability rules always produce under-deterrence (except in the range in which

---

129. Game theorists distinguish between two types of equilibria: equilibria in pure strategies and equilibria in mixed strategies. For our purposes, a pure strategy is simply the choice of a level of care. In contrast, under a mixed strategy, an actor would not pick a single level of care, but instead would pick a probability distribution over multiple levels of care. On mixed strategies, see R. LUCE & H. RAIFFA, GAMES AND DECISIONS 70-71 (1957).
there is no equilibrium in pure strategies). Full liability, unitary share rules also generally produce under-deterrence. The exception here is a range in which the standard of care is stringent but sufficiently close to the social optimum. In this case, they produce over-deterrence (for these rules too, there is a range in which there is no equilibrium in pure strategies).

It is only for full liability rules in the range of over-deterrence that fractional share rules might be preferred to unitary share rules; even there, however, they might lead to too much under-deterrence and therefore not be desirable. In all other instances, and for all partial liability rules, unitary share rules are preferable.\(^{130}\)

Figure IV also reveals that setting the standard of care too stringently might lead to greater under-deterrence than setting it too leniently. Under full liability rules, if the standard of care is set in the range between 0 and \(a_1\), the actors will choose to dump the strict liability amount, whereas if the standard of care is set between the social optimum and the strict liability level, they will dump the lesser amount prescribed by this standard of care. The same result holds, albeit in different ranges, for partial liability rules.

**FIGURE IV: NON-OPTIMAL BUT CERTAIN STANDARDS OF CARE**

<table>
<thead>
<tr>
<th>Standard of Care</th>
<th>Equilibrium Levels of Care†</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full Liability</td>
</tr>
<tr>
<td>Stringent</td>
<td></td>
</tr>
<tr>
<td>0 - (a_1)</td>
<td>(x_S) (u)</td>
</tr>
<tr>
<td>(a_1 - a_2)</td>
<td>no eq.</td>
</tr>
<tr>
<td>(a_2 - a_3)</td>
<td>(c) (o)</td>
</tr>
<tr>
<td>(a_3 - x^*)</td>
<td>(c) (o)</td>
</tr>
<tr>
<td>Lenient</td>
<td></td>
</tr>
<tr>
<td>(x^* - x_S)</td>
<td>(c) (u)</td>
</tr>
<tr>
<td>(x_S - \infty)</td>
<td>(c) (u)</td>
</tr>
</tbody>
</table>

† "u" denotes under-deterrence; "o" denotes over-deterrence; "no eq." denotes no equilibrium in pure strategies.

130. Our analysis of optimal standards with uncertainty and non-optimal standards without uncertainty, which we have applied primarily to negligence rules, applies also to the efficient strict liability rules that we propose in this Article.
In conclusion, we return to the question of when joint and several liability is desirable, from an efficiency perspective, under negligence rules. We showed in Section III that, when the standard of care is set optimally without uncertainty, joint and several liability is necessary to induce the efficient outcome. We show in this Section that joint and several liability is preferable to non-joint (several only) liability except in two special circumstances. Opponents of joint and several liability must defend the existence of optimal but uncertain standards and argue that the probability distribution that describes the standard of care is neither tight enough to recommend the use of partial liability, unitary share rules nor disperse enough to call for full liability, unitary share rules. Alternatively, they must point to non-optimal but certain standards under full liability rules, and argue that the standard of care is too stringent but not so stringent as to preclude an equilibrium in which actors meet the standard of care, and that the under-deterrence that might be caused by non-joint liability is no more undesirable than the over-deterrence of joint and several liability. The opponents of joint and several liability within the tort reform movement have not met this burden.

CONCLUSION

Our analysis has two principal conclusions. First, we study negligence rules with the standards of care set at the socially optimal levels and show that any unitary share rule will be efficient, but that other rules will not. Second, we study strict liability and show that none of the existing rules of apportionment are efficient, but that one can design efficient apportionment rules.

These results are analogous to those reached in the economic literature on the apportionment of losses between an “injurer” and a “victim,” each of whose actions influence the expected cost of an accident. In this literature, the legal rule assigns the loss to one party or the other, just as the rule of apportionment does for multiple tortfeasors. In the injurer/victim context, a rule of comparative negligence provides the closest analogy to a rule of apportionment under negligence for multiple injurers.

Recent studies of comparative negligence have shown that when the standards of care are set optimally, both victim and injurer will adopt the socially optimal levels of care. This argument parallels the argument

131. We have indicated that, for traditional strict liability rules, given the assumptions of this Article (in particular, the lack of insolvency), the choice between joint and several liability on the one hand and non-joint (several only) liability on the other has no efficiency consequences. See supra text accompanying notes 98–99.
132. See supra note 73.
133. In contrast, under the model that we use to study the problem of joint tortfeasors, the cost of the accident is independent of the victim’s actions.
that establishes an identical result for the rules of pure negligence, negligence with contributory negligence, strict liability with contributory negligence, and strict liability with dual contributory negligence. The structure of each of these rules identifies a "default" bearer of liability and gives the other actor an opportunity to avoid all liability by meeting her standard of care. In equilibrium, the injurer as well as the victim "sees" the full social cost of her decision. The default bearer of liability sees it because, in equilibrium, she in fact bears the loss. The other actor sees the full social cost of her decision because, in equilibrium, a marginal decrease in her level of care would make her negligent and shift the entire loss to her.

Under negligence, unitary share rules of apportionment function identically. The victim serves as the default bearer of liability. The negligence rule assures that any injurer can avoid all liability by meeting the standard of care. A unitary share rule assures that, if a single injurer deviates from her standard of care (while all others meet theirs), then a cost at least equal to the full increase in the loss will fall on her alone. Thus, all injurers "see" the full loss in equilibrium.

Similarly, under the apportionment rules that we propose, in the strict liability context, if all actors are at the optimum, an actor contemplating a departure from the optimum faces a cost that is greater than the full increase in the loss. This actor therefore also "sees" the full loss in equilibrium.

Conversely, the existing apportionment rules under strict liability are inefficient for the same reason that strict liability is inefficient in the standard victim/injurer context. As losses are purely economic, the victim

\[\text{STUD. 49, 68-71 (1985), show that, if the standards of care are set "optimally," both injurer and victim will adhere to their standards of care; hence, the victim will bear the entire loss. ("Optimality" refers to the levels of care that minimize the sum of accident prevention costs and expected losses from the accident.)}\]

Neither article analyzes cases in which the standard of care is set non-optimally and without uncertainty. These cases are more complex. First, fix the victim’s standard of care at the optimal level. Now increase the injurer’s standard of care above the optimal level for the injurer. The injurer’s optimal response, given that the victim continues to meet his standard of care, is, over some range of values, to set her level of care at the optimal level. The victim’s optimal response to this decision, however, is to set his level of care at less than the optimal level. Faced with this decision, the injurer might well choose a level of care higher than the optimum, in which case the victim might best respond with a different level of care. Hence, no equilibrium in pure strategies may exist.

135. Under a scheme of pure negligence, the injurer is liable if she violates her standard of care, regardless of the victim’s level of care. For negligence with contributory negligence, the injurer is liable if she violates her standard of care, except when the victim violates his standard of care. Under strict liability with contributory negligence, the injurer is liable regardless of her level of care unless the victim violates his standard of care. Finally, for strict liability with dual contributory negligence, the injurer is liable except when the victim violates his standard of care and the injurer does not violate her standard of care. Mathematical descriptions of these rules are provided in Brown, supra note 30.

136. Under partial liability rules, the actor will have to bear the full increase in the loss. Under full liability rules, she will have to bear this increase plus the loss that results when the standard of care is met.
has no incentive to take care and hence too many accidents will occur. In the multiple actor context, each injurer under strict liability bears some, but not all, of the loss. At the socially optimal level of care, the marginal benefit of expected loss reduction exceeds the costs of producing that reduction. Consequently, each injurer takes too little care (or, in our model, dumps too much waste). Thus, the analogies that we present in this conclusion serve to explain the connection between what are, in fact, closely related areas of tort law.\textsuperscript{139}

\begin{flushright}
137. \textit{See} S. Shavell, \textit{supra} note 5, at 11. Fractional share rules under negligence fail for the same reason.


\end{flushright}
We now define our model more formally, and set forth the notation that we will use. Suppose that there is a set $N$ of actors engaging in an activity. Each actor $i \in N$ adopts a level of care $x_i \in (0, \infty)$. We employ the convention that a higher $x_i$ represents less care (or more carelessness).

Each actor $i$ derives a benefit, $B_i(x_i)$, from choosing to engage in the activity with the level of care $x_i$. We assume that each benefit function is concave. The actors' choices $x_i$ cause a social loss, $L(x)$, where $x$ is the vector whose ith component is $x_i$. We assume that $L(x)$ is convex, with $L(0) \geq 0$ and $\partial L(x)/\partial x_i > 0$ for $i \in N$. We also assume that each actor has sufficient assets to bear the full liability for the social loss, thus avoiding the complicated analysis of the effect of insolvency on rules of contribution. Moreover, we follow the prior literature in considering solely an impersonal or economic social loss and in excluding personal injuries from the analysis.

The social objective function, $W(x)$, is defined by $W(x) = \sum B_i(x_i) - L(x)$.\textsuperscript{140} It is the sum of benefits that the actors derive from the activity minus the social loss that they cause. Let $x^*$ be the vector that maximizes the social objective function. To insure a unique interior maximum, we require that the $B_i$ be strictly concave, that $L$ be strictly convex, and that $B_i'(0) > \partial L(0)/\partial x_i$, where $\partial L(0)/\partial x_i$ denotes the partial derivative of $L(x)$ evaluated at $x = 0$.\textsuperscript{141}

A rule of negligence is defined as a vector $c$, where the ith component, $c_i$, of the vector is the standard of care for actor $i$. To meet the standard of care actor $i$ must set $x_i \leq c_i$; if she does so, under a negligence rule, she will bear no portion of the social loss. Let $M = \{ i | x_i > c_i \}$ be the set of negligent actors. Under this formulation, a strict liability rule is simply a special case of a negligence rule with $c = 0$.

A rule of apportionment consists of a vector of share functions, $f_i(x,c)$, with $0 \leq f_i(x,c) \leq 1$, and an allocation function, $A(x,c)$, such that for each actor $i$, $f_i(x,c)$ is the actor's fractional share of the total amount of damage allocated to injurers; we also define actor $i$'s share of liability as the product $f_i(x,c)A(x,c)$ of the actor's share function and the allocation function. We consider only allocation functions of the form $A(x,c) = L(x) - K(c)$, where $K(c)$ is constant in $x$ and depends only on $c$. Since $c$ affects

\textsuperscript{140} Where the limits of summation are not indicated, the summation is over the set $i \in N$. See infra note 144.

\textsuperscript{141} Our social objective function is equivalent to that used by Landes and Posner. See Landes & Posner, supra note 5, at 521-22. Their function minimizes costs rather than maximizes benefits. Consequently, they interpret $x_i$ as an actor's level of care (with higher levels of $x_i$ leading to smaller losses), whereas we interpret it as an actor's level of carelessness (so that lower levels of $x_i$ lead to smaller losses). Higher levels of care, in Landes and Posner's model, carry with them higher costs (or negative benefits).
A(x,c) only through this constant term, we shall suppress the dependence of A(x,c) on c and write the allocation function simply as A(x). Accordingly, we may define the five main types of liability rules.

**Definition:** A rule of apportionment is a **full liability rule** if and only if 
\[ A(x) = L(x). \]

**Definition:** A rule of apportionment is a **partial liability rule** if and only if 
\[ A(x) \leq L(x) \] for all x, with strict inequality for some x.\(^{142}\)

**Definition:** A rule of apportionment is a **(common law) partial liability rule** if and only if 
\[ A(x) = \max \left\{ L(x) - L(c), 0 \right\}. \] \(^{143}\)

**Definition:** A rule of apportionment is a **unitary share rule** if and only if 
\[ \sum_{M} f_i(x,c) = 1 \] if M is non-empty. \(^{144}\)

**Definition:** A rule of apportionment is a **fractional share rule** if and only if 
\[ \sum_{M} f_i(x,c) < 1 \] for some M (and equal to 1 otherwise). The text (and the figures in the text) require a strict inequality for all M different from N.

Consider a negligence rule with standard of care c and a rule of apportionment that assigns \( f_i(x,c)A(x) \) to actor i. The ith **payoff function** is defined by

\[
V_i(x,c) = \begin{cases} 
B_i(x_i) - f_i(x,c)A(x) & \text{for } x_i > c_i \\
B_i(x_i) & \text{otherwise}
\end{cases}
\]

The n payoff functions and n strategy spaces \((0, \infty)\) define a game, the Nash equilibrium of which we study. A vector \( y^* \) is a Nash equilibrium of the game if and only if for each i, 
\[ V_i(y_i, y_{-i}^*) \leq V_i(y^*), \] where \((y_i, y_{-i}^*)\) is the n-vector for which the ith component is \( y_i \) and the jth component is \( y_j^* \) for \( j \neq i \).

With this background, we may consider the following propositions.

**Proposition 1:** Consider a negligence rule in which the standard of care for each i is set at \( x_i^* \). Let \( \{ f(x,x^*), A(x) \} \) be a rule of apportionment that satisfies

(a) \( A(x) \leq L(x) \);

(b) \( \partial A(x)/\partial x_i = \partial L(x)/\partial x_i \) for each i; and

\[^{142}\] Because we deal with risk-neutral actors (and ignore insolvency), we have elided distinctions between ex ante and ex post losses. Our loss function \( L(x) \) is actually an ex ante loss function. More generally, \( L(x) = p(x)D(x) \), where \( p(x) \) is the probability of the loss occurring given choice \( x \), and \( D(x) \) is the ex post loss suffered (in the event of an accident) given \( x \). To pursue the ex ante/ex post distinction, we should write \( A(x) = p(x)C(x) \), where the actor pays \( C(x) \) in the event of an accident. The "no-punitive-damages" condition of the text would then require \( C(x) \leq D(x) \).

\[^{143}\] Full liability and partial liability rules do not exhaust the class of possible rules. There are also **punitive liability rules** in which \( A(x) \geq L(x) \) with strict inequality for some \( x \). We do not consider punitive liability either in the text or in this Appendix. Similarly, there are many partial liability rules with \( A(x,c) \leq L(x) \) for all \( x \) and \( c \) that have more complex dependence on \( c \) than the (common law) partial liability rules considered in the rest of the Appendix and throughout the text.

\[^{144}\] A summation over the set \( i \in M \) is indicated as \( \sum_M \); the same convention is used for summations over other sets, except as indicated *supra* note 140.
(c) $\sum_{M} f_i(x, x^*) = 1$ where $M$ is non-empty. Then $x^*$ is the unique Nash equilibrium.

Note that condition (b) is satisfied by any full liability rule and, for the purposes of the proof, any (common law) partial liability rule. The (common law) partial liability rules discussed in the text set $K(c) = L(x^*)$ when the standard of care is set optimally. Condition (c) requires that the rule be unitary. Proposition 1 therefore states that any unitary rule that sets the standard of care optimally induces efficiency.

Proof: Optimality of $x^*$ implies that for each $i$, $x_i^*$ satisfies $B_i(x_i) - \partial L(x)/\partial x_i = 0$. Given condition (b), $x^*$ also maximizes $\sum B_i(x_i) - A(x)$. It is therefore clear that $x^*$ is a Nash equilibrium. For suppose that all actors except $i$ have chosen $x_j = x_j^*$. Then $i$, who must bear the entire loss $A(x)$, chooses $x_i$ to maximize

$$V_i((x_j, x_{i^*}), x^*) = \begin{cases} B_i(x_i) - A(x_i, x_{i^*}) & \text{for } x_i > x_i^* \\ B_i(x_i) & \text{otherwise} \end{cases}$$

By definition of the optimum, $x_i^*$ maximizes

$$B_i(x_i) - A(x_i, x_{i^*})$$

Thus $i$ will choose $x_i = x_i^*$.

Suppose another Nash equilibrium $x'$ exists; given the above, at $x'$, $m$ actors (with $m > 1$) must be negligent and $(n-m)$ non-negligent. Thus for $j$ in $N-M$, $x_j' = x_j^*$, and for $j$ in $M$, $x_j'$ maximizes

$$V_j((x_j, x_{j^*}), x^*) = B_j(x_j) - f_j((x_j, x_{j^*}), x^*) A(x_j, x_{j'})$$

with $V_j(x', x^*) \geq B_j(x_i^*)$. Thus

$$V(x', x^*) = \sum V_j(x_j') = \sum_{M} B_j(x_j') + \sum_{N-M} B_j(x_j^*) - A(x')$$

$$\geq \sum_{M} B_j(x_j^*)$$

$$> \sum_{M} B_j(x_j^*) - L(x^*)$$

which violates the optimality of $x^*$. Q.E.D.

Corollary 1: Consider a rule of no-contribution under which each actor's expected share is common knowledge in that such shares are publicly known. If the legal rule sets each actor's standard of care at $x_i^*$, then the unique Nash equilibrium has $x_i = x_i^*$ for all $i$.

Proof: If expected shares are common knowledge, then the negligent actors know that they will bear the entire loss; thus each believes that she faces the maximization problem described in the proposition. Q.E.D.

The common knowledge assumption of the corollary states that the sum of the parties' expected shares of the loss is 1. Of course, if the sum of

145. The structure of this proof of uniqueness appears in Landes & Posner, supra note 5, at 523-24, as a proof of existence.
expected shares exceeds 1, then the unique Nash equilibrium remains \( x_i = x_i^* \) for all \( i \); indeed, no actor will choose a level of care more stringent than the standard of care because doing so would entail foregoing benefits with no compensating decrease in liability.

\[
* * * *
\]

For a wide class of (common law) partial liability, fractional share rules, efficiency never results. That is,

**Proposition 2:** Suppose a rule of negligence with standards of care set at the social optimum \( x^* \) prevails. Let \( \{ f(x,x^*), A(x) \} \) be a rule of apportionment that satisfies

(a) \( \sum_M f_i(x,x^*) < 1 \) for every set \( M \) (different than \( N \)) of negligent actors; and

(b) \( A(x) = L(x) - L(x^*) \).

Then,

1. \( x^* \) is not a Nash equilibrium;
2. Let \( y \) be a Nash equilibrium in pure or mixed strategies; actor \( i \)'s strategy \( y_i \) is described by a probability distribution function \( G_i(x_i) \) on the interval \((0,\infty)\). For every \( i \), \( G_i(x_i) = 0 \) for \( x_i < x_i^* \), and there exists at least one actor \( k \) for which \( G_k(x_k^*) < 1 \).

Condition (a) conforms to the definition of fractional share rules used in the text. Condition (b) identifies a (common law) partial liability rule.

**Proof:** Consider actor \( i \)'s payoff function when all other actors \( j \) choose \( x_j^* \):

\[
V_i((x_i,x_i^*),x^*) = \begin{cases} 
B_i(x_i) - f_i((x_i,x_i^*),x^*)A(x_i,x_i^*) & \text{for } x_i > x_i^* \\
B_i(x_i) & \text{otherwise}
\end{cases}
\]

The first-order condition, conditional on being negligent, is

\[
f_i \frac{\partial A(x_i,x_i^*)}{\partial x_i} + A(x_i,x_i^*) \frac{\partial f_i}{\partial x_i} = B_i'(x_i)
\]

Evaluating the left hand side at \( x_i^* \) yields

\[
f_i \frac{\partial A(x^*)}{\partial x_i} + A(x^*) \frac{\partial f_i}{\partial x_i}
\]

But condition (a) states that \( f_i < 1 \), and condition (b) implies that \( A(x^*) = 0 \). Thus

\[
f_i \frac{\partial A(x^*)}{\partial x_i} + A(x^*) \frac{\partial f_i}{\partial x_i} < \frac{\partial A(x^*)}{\partial x_i} = B_i'(x_i^*)
\]

It follows that \( x_i^* \) does not satisfy the first-order condition for actor \( i \). This proves part (1). Part (2) follows from the fact that, in equilibrium,
no actor will ever choose \( x_i < x_i^* \) because doing so would entail foregoing benefits with no compensating decrease in liability. Q.E.D.

Note that condition (a) is not necessary for the proof in that it requires that the negligent parties (in the set \( M \)) never bear the entire loss \( A(x) \) unless \( M = N \). To show that \( x^* \) is not an equilibrium we required only that \( f_i < 1 \) for the set \( M = \{i\} \) for some \( i \), not for every \( i \). That assumption is necessary and sufficient to insure that \( x^* \) is not a Nash equilibrium.

** **

Whether full liability, fractional share rules induce the efficient result depends on the benefit and loss functions and on the number of actors. That is,

**Proposition 3:** Suppose a rule of negligence with standards of care set at the social optimum \( x^* \) prevails. Let \( \{ f(x, x^*), A(x) \} \) be a rule of apportionment that satisfies

(a) \( \sum_M f_i(x, x^*) < 1 \) for every set \( M \) (different than \( N \)) of negligent actors; and

(b) \( A(x) = L(x) \).

Then whether \( x^* \) is a Nash equilibrium depends on \( B_i(x_i), L(x), \) and \( n \).

**Proof:** We prove this proposition by providing an example in which \( x^* \) is a Nash equilibrium and a counterexample in which it is not. Consider a full liability, fractional, fixed share (per capita) rule (Rule 3). Let there be \( n \) actors; \( L(x) = X^r \), where \( X = \sum x_i \); and for each actor \( i \), let \( B_i(x_i) = x_i \). Then the optimum total production \( X^* = (1/r)^{(r-1)} \), and the standard of care can be set at \( x_i^* = (1/n)X^* \) for all \( i \).\(^{146}\) Suppose that \((n-1)\) actors meet the standard of care; the question then is whether it would be rational for the remaining actor, actor \( i \), to be negligent. If actor \( i \) meets the standard of care, she will not be liable for any of the damage. If, in contrast, she is negligent, she will be liable for \( 1/n \) of the total damage. Actor \( i \)'s payoff function is given by

\[
V_i((x_i, x_i^*), x^*) = \begin{cases} 
  x_i - (1/n) \left\{ x_i + [(n-1)/n](1/r)^{1/(r-1)} \right\} & \text{for } x_i > x_i^* \\
  x_i & \text{otherwise}
\end{cases}
\]

The first-order condition for \( i \)'s maximization problem, conditional on her being negligent, is

\(^{146}\) Since the benefit functions are identical and linear, it does not matter how \( X^* \) is divided among the actors.
Solving for $x_i$ we have

$$x_i = \frac{(n/r)l}{r-1} - \left[\frac{(n-1)/n}{1/r}\right] \frac{l}{r-1}$$

It follows that $x^*$ is not a Nash equilibrium if $i$'s payoff from being negligent is greater than her payoff from being non-negligent, that is, if

$$\frac{(n/r)l}{r-1} - \left[\frac{(n-1)/n}{1/r}\right] \frac{l}{r-1} > 0$$

which is equivalent to

$$\frac{(n/r)}{r-1} - \left[\frac{1/r}{1/r}\right] \frac{l}{r-1} > 0$$

which holds if and only if

$$\left[\frac{n/r}{1/r}\right] \frac{l}{r-1} > 0$$

For $n = 2$, this expression holds for $1 < r < 2$; for larger $n$, it holds for all $r > 1$ Q.E.D.

Recall that we may write strict liability as a negligence rule which sets the standard of care at 0 for each $i$.

**Proposition 4:** Suppose a rule of strict liability governs. Let

{ $f(x,0), A(x)$ } be a rule of apportionment that satisfies

(a) all $f_i(x,0) = q_i$, or all $f_i(x,0) = x_i/X$, where $X = \Sigma x_i$;

(b) $A(x) = L(x)$; and

(c) $\Sigma f_i(x,0) = 1$.

Then, (1) $x^*$ is not a Nash equilibrium; and (2) if $A(x) = \alpha(X)$ for some function $\alpha$, then any Nash equilibrium $x'$ has the property $X' > X^*$.

**Proof:** Consider actor $i$'s payoff function when all other actors $j$ choose $x_j^*$:

$$V_i((x_i,x_i^*),0) = B_i(x_i) - f_i((x_i,x_i^*),0)A(x_i,x_i^*)$$

The first-order condition is

$$f_i \frac{\partial A(x_i,x_i^*)}{\partial x_i} + A(x_i,x_i^*) \frac{\partial f_i}{\partial x_i} = B'_i(x_i)$$

147. That is, $A(x)$ depends only on $\Sigma x_i$. 
For \( f_i = q_i \) this becomes
\[
q_i \frac{\partial A(x_i, x_i^*)}{\partial x_i} = B_i'(x_i)
\]
Evaluating the left-hand side at \( x_i^* \) yields
\[
q_i \frac{\partial A(x^*)}{\partial x_i}
\]
Condition (c) implies that there is at least one \( i \), say \( k \), such that \( q_k < 1 \).
For actor \( k \), \( x_k^* \) does not satisfy the first-order condition as
\[
q_k \frac{\partial A(x^*)}{\partial x_k} < \frac{\partial A(x^*)}{\partial x_k} = B_k'(x_k^*)
\]
This proves part (1) for \( f_i = q_i \).
For \( f_i = x_i/X \), the first-order condition is
\[
\left[ x_i/(x_i + \Sigma_{j \neq i} x_j^*) \right] \frac{\partial A(x_i, x_i^*)}{\partial x_i} + \frac{A(x_i, x_i^*)}{X^*} \left( \Sigma_{j \neq i} x_j^* \right) = B_i'(x_i)
\]
Evaluating the left-hand side at \( x_i^* \) yields
\[
\left( x_i^*/X^* \right) \frac{\partial A(x^*)}{\partial x_i} + A(x^*) \left( \Sigma_{j \neq i} x_j^* \right)/X^{*2}
\]
But given the strict convexity of \( A(x) \), \( A(x^*)/X^* < \partial A(x^*)/\partial x_i \). Thus
\[
\left( x_i^*/X^* \right) \frac{\partial A(x^*)}{\partial x_i} + A(x^*) \left( \Sigma_{j \neq i} x_j^* \right)/X^{*2} < \frac{\partial A(x^*)}{\partial x_i} = B_i'(x_i^*)
\]
It therefore follows that \( x_i^* \) does not satisfy the first-order condition. This proves part (1) for \( f_i = x_i/X \).
Assume that, for \( f_i = q_i \), there is an equilibrium \( x' \) for which \( X' \leq X^* \). Given the first-order condition,
\[
q_i \alpha'(X') = B_i'(x_i')
\]
But given the convexity of \( A(x) \),
\[
B_i'(x_i') = q_i \alpha'(X') \leq q_i \alpha'(X^*) \leq \alpha'(X^*) = B_i'(x_i^*)
\]
Since \( B_i(x_i) \) is strictly concave, it follows that \( x_i' \geq x_i^* \) for all \( i \). Moreover, for some \( k \), \( q_k < 1 \), from which it follows that \( x_k' > x_k^* \). These inequalities contradict the assumption that \( X' \leq X^* \). This proves part (2) for \( f_i = q_i \). There is a similar proof for \( f_i = x_i/X \). Q.E.D.
Corollary 1: $x^*$ is not a Nash equilibrium under a rule of strict liability and a no-contribution rule.

Proof: A no-contribution rule is simply a fixed share rule. Q.E.D.

Corollary 2: A rule of apportionment and a rule of no-contribution will be equivalent under strict liability if and only if $f_i$ equals i's expected share of liability under no-contribution.

Proof: Each actor's maximization problem is a function of $f_i$. Q.E.D.

* * *

One can design a rule of apportionment that induces efficiency even under strict liability. Proposition 1 strongly suggests the form that rule of apportionment should have. Strict liability requires that the actors bear the entire loss $L(x)$ but the rule of apportionment permits us to make the assignment of the loss depend on meeting "standards of apportionment." That is, we may essentially use the rule of apportionment to replicate, under strict liability, the incentive scheme generated by a negligence regime.

The statement and proof of Proposition 5 require some additional notation. Let $M^* = \{ i \mid x_i > x_i^* \}$ where $x_i^*$, as before, indicates the choice of $x_i$ that maximizes social welfare. With this definition, we may state and prove the following.

Proposition 5: Suppose strict liability governs the actors' actions. Let $(f(x,0), A(x))$ be a rule of apportionment that satisfies

(a) $A(x) = L(x)$;

(b) for all $x$ such that $M^*$ is empty,

(i) $\sum f_i(x,0) = 1$;

(ii) for each $i$, on the interval $(0,x_i^*]$, $x_i^*$ maximizes

$$B_i(x_i) - f_i(x,0)A(x);$$

and

(c) for all $x$ such that $M^*$ is non-empty, $\sum f_i(x,0) = 1$.

Then $x^*$ is the unique Nash equilibrium.

Proof: We first show that $x^*$ is an equilibrium. So suppose that all actors except $i$ have chosen $x_j = x_j^*$. Then by condition (c), if $i$ is negligent, she will bear the entire loss $A(x_i^*,x_i^*)$. Thus she would choose $x_i$ to maximize

$$B_i(x_i) - A(x_i,x_i^*)$$

But $x_i^*$ maximizes this expression; $i$ would thus adopt $x_i^*$. The proof of uniqueness is essentially the same as that given in Proposition 1. Q.E.D.

Note that condition (c) creates discontinuities at $x^*$ that replicate, through the rule of apportionment, the incentives generated by a full liability, unitary share rule under negligence with the standards of care set
optimally. In turn, condition (b)(ii) insures that $x^*$ is the unique equilibrium with $M^*$ empty.\textsuperscript{148}

Proposition 5 does not necessarily capture all the rules of apportionment that induce efficiency under strict liability. Appropriate incentives require only that the change in $f_i$ at $x^*$ be sufficiently dramatic to induce the choice of $x_i^*$. Thus some continuous and perhaps even differentiable set of $f_i$ might induce efficiency.

* * * *

We now turn to the case of uncertainty.

Proposition 6: Suppose there are two actors. Consider a negligence rule with the standard of care for actor 1 drawn from a probability distribution with mean $x^*$ and the standard of care for actor 2 drawn from a probability distribution with mean $y^*$. Let the rule of apportionment be $A(x,y) = L(x,y) - L(x^*,y^*)$. Then,

(a) the partial liability rule $A(x,y)$ always yields under-deterrence; and

(b) a partial liability, fractional share rule under-deters more than a partial liability, unitary share rule.

Proof: The logic of the proof follows that used by Craswell and Calfee in their analysis of the single actor case.\textsuperscript{149}

Let $II(x \mid y^*)$ be one actor's expected profit given that the other actor is operating at the social optimum. Let $F(x)$ be the probability that this actor is held liable given that she has adopted level of care $x$, and let $G(y)$ be the probability that the other actor is held liable given that she has adopted level of care $y$.\textsuperscript{150} For a unitary, fixed share (per capita) rule,

\textsuperscript{148} In the absence of condition (b)(ii), the $f_i$ might be such that a set of discontinuities occurs at some point $x' < x^*$; that is, at a point $x'$ each of whose components is less than the corresponding component of $x^*$. These $f_i$ would reproduce a full liability, unitary share rule under negligence with the standards of care set non-optimally; as shown below, such a rule need not induce efficiency.

Also suppose that the $f_i$ have discontinuities at both $x'$ and $x^*$. Under negligence, there would be no multiplicity of equilibria because, for $x' < x^*$, the injurers bear none of the liability. Under strict liability, in contrast, there could be multiple equilibria because the injurers bear all the liability.

Note that the possibility of multiple equilibria is not ruled out simply by requiring that $f_i$ be continuous, as there may be such multiple equilibria with continuous functions if $f_i$ is rising sufficiently rapidly. Both fixed share rules and proportional share rules satisfy condition (b)(ii).

\textsuperscript{149} Craswell & Calfee, supra note 117.

\textsuperscript{150} The probability distribution functions $F$ and $G$ are derived from the respective distributions over the standards of care.
\[ \Pi(x, y^*) = B(x) - F(x) \left\{ [1 - G(y^*)]A(x, y^*) + G(y^*)A(x, y^*)/2 \right\} \\
= B(x) - F(x)A(x, y^*)[1 - G(y^*)/2] \]

Then
\[ \frac{\partial \Pi(x, y^*)}{\partial x} = B'(x) - [1 - G(y^*)/2] \left\{ F(x) \frac{\partial A(x, y^*)}{\partial x} + F'(x)A(x, y^*) \right\} \]

But \( B'(x) = \frac{\partial A(x, y^*)}{\partial x} \). Thus
\[ \frac{\partial \Pi(x, y^*)}{\partial x} = \left\{ 1 - [1 - G(y^*)/2] F(x) \frac{\partial A(x, y^*)}{\partial x} - [1 - G(y^*)/2]F'(x)A(x, y^*) \right\} \]

If \( A(x, y^*) = 0 \), as it is for a (common law) partial liability rule, then the second term is zero, the first term is positive, and there are incentives to undercomply.

For a fractional, fixed share (per capita) rule,
\[ \Pi(x, y^*) = B(x) - F(x)A(x, y^*)/2 \]

Then
\[ \frac{\partial \Pi(x, y^*)}{\partial x} = [1 - F(x)/2] \frac{\partial A(x, y^*)}{\partial x} - F'(x)A(x, y^*)/2 \]

If \( A(x, y^*) = 0 \), as it is for a (common law) partial liability rule, then the second term is zero, the first term is positive, and there are incentives to undercomply. Comparing the expressions for the unitary and fractional rules, the incentive to undercomply is smaller for unitary share rules than for fractional share rules. Q.E.D.

* * * *

Finally, we present an example that illustrates the responses of two identical actors, to standards of care that are certain but are set too stringently. We perform the analysis for unitary, fixed share (per capita) rules.

151. Our formulation of the model is better suited to an interpretation in which the standard of care is known for certain but the court makes errors of fact-finding. Under the interpretation in the text, where \( E(c_1) = x^* \) and \( E(c_2) = y^* \), it would seem more appropriate to define the partial liability rule as a function not of the mean standard of care but of the realized standard of care (that is, to set \( A(x, y, c_1, c_2) = L(x, y) - L(c_1, c_2) \) where \( c_1 \) is the standard of care imposed on actor 1 and \( c_2 \) the standard of care imposed on actor 2). The analysis of this model is more complex as actor 1 is concerned with \( E[A(x, y)] = L(x, y) - E[L(c_1, c_2)] \). Each actor thus apparently faces an additional incentive to undercomply as, by the strict convexity of \( L \) and Jensen’s inequality, \( E[L(c_1, c_2)] \geq L(x^*, y^*) \) and therefore \( E[A(x, y)] < L(x, y) - L(x^*, y^*) \). On Jensen’s inequality, see M.H. deGroot, Optimal Statistical Decisions 97 (1970).
Let \( L(x) = X^r \), where \( r > 1 \); and let \( B_i(x_i) = x_i \). Recall that \( X = \sum x_i \). The optimum total production \( X^* = (1/r)^{1/(r-1)} \), and the optimal standard of care can be set at \( x_i^* = x^* = 1/2(1/r)^{1/(r-1)} \). Throughout this example, both actors face the same standard of care.\(^{152}\)

For ease of exposition, we define the equilibrium level of care under a strict liability, fixed share (per capita) rule. Under such a rule, each actor would maximize

\[
x_i - 1/2X^r.
\]

Let \( (x_s, x_a) \) be the equilibrium level of care. Then \( x_s = 1/2(2/r)^{1/(r-1)} \). Even though in this example we are studying the behavior of negligence rules, we define the equilibrium under strict liability because if both actors decide to violate the standard of care, they will behave as if they were governed by a strict liability rule.

We consider first how a full liability rule under negligence performs under standards of care that are more stringent than the social optimum. There is an equilibrium at \( (x_s, x_a) \) where the standard of care is set in the range \( 0 \leq c \leq a_1 \), where \( a_1 \) is defined by

\[
a_1 = x_s - 1/2(2x_s)^r
\]

The value \( a_1 \) is the highest level of the standard of care for which, given that one actor is operating at \( x_s \), the other actor will also choose \( x_s \) rather than meet the standard of care. The left hand side of the equation gives one actor's benefit from meeting the standard of care set at \( a_1 \), and the right hand side is the benefit to that actor from operating at \( x_s \), given that the other actor is also operating at \( x_s \). Substituting for \( x_s \) in the preceding equation we obtain

\[
a_1 = (2/r)^{1/(r-1)}(r - 2)/(2r)
\]

Note that \( a_1 > 0 \) for all \( r > 2 \), and that \( a_1/x^* = (2)^{1/(r-1)}(1 - 2/r) < 1 \) for all \( r \). Thus \( a_1 < x^* \). It follows that if the standard of care is set sufficiently below \( x^* \), a negligence rule behaves like a strict liability rule.

There is an equilibrium at \( (c, c) \) where the standard of care is set in the range \( a_2 \leq c \leq x^* \) for \( a_2 = a_m - (a_m + a_2)^r \geq 0 \), where \( a_m = \text{argmax} [x - (x + a_2)^r] \). The value \( a_2 \) is the lowest level of the standard of care for which, given that one actor is meeting the standard of care, the other actor will also meet the standard of care. The value \( a_m \) is the optimal level of care for an actor who has chosen to violate the standard of care set at \( a_2 \), given that the other actor has chosen to meet that standard of care. Solving first for \( a_m \) we obtain

\[152. \text{ See supra note 146.}\]
\[ 1 = r(a_m + a_2)^{r-1} \]
\[ a_m = \frac{(1/r)^{1/(r-1)}}{a_2} \]

It follows that
\[ a_2 = \frac{(1/r)^{1/(r-1)}}{a_2} - \frac{(1/r)^{r/(r-1)}}{a_2} \]
\[ a_2 = \frac{(1/r)^{1/(r-1)}(r - 1)/(2r)}{a_2} \]

Note that \( a_2 > 0 \) for all \( r > 1 \). Note also that \( a_1 < a_2 < x^* \) for all \( r \). Thus if the standard of care is sufficiently close to \( x^* \), the actors will meet the standard of care. Finally, note that where the standard of care is set in the range \( a_1 < c < a_2 \), there is no equilibrium in pure strategies. Where the standard of care is in this range, it is neither sufficiently close to the socially optimal level for an equilibrium to exist in which each actor meets the standard of care, nor sufficiently far from the socially optimal level for an equilibrium to exist in which each actor operates at the strict liability level.

We now turn to the examination of (common law) partial liability rules. There is an equilibrium at \((x_s, x_s)\) where the standard of care is set in the range \( 0 \leq c \leq a_3 \), where \( a_3 \) is defined by
\[ a_3 = x_s - \frac{[(2x_s)^r - (2a_3)^r]/2}{a_3} \]
\[ a_3 - \frac{1}{2}(2a_3)^r = x_s - \frac{1}{2}(2x_s)^r \]

The value \( a_3 \) is the highest level of the standard of care for which, given that one actor is operating at \( x_s \), the other actor will also choose \( x_s \) rather than meet the standard of care. As in the case of full liability rules, a negligence rule behaves like a strict liability rule if the standard of care is set too stringently. Note that where the standard of care is set in the range \( a_3 < c < x^* \), there is no equilibrium in pure strategies.\(^{153}\)

Finally, to compare full liability and (common law) partial liability rules, we need to determine the relationship between \( a_2 \) and \( a_3 \). Substituting for \( a_2 \) and \( x_s \), it follows that
\[ a_2 < x_s - \frac{[(2x_s)^r - (2a_2)^r]}{2} \text{ for all } r > 2 \]

Thus \( a_2 < a_3 \) for all \( r > 2 \). As a result, where the standard of care is set in the range \( a_2 < c < a_3 \), a (common law) partial liability rule produces an equilibrium at \((x_s, x_s)\), causing under-deterrence, whereas a full liability rule produces an equilibrium at \((c, c)\), causing over-deterrence.

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153. See supra text accompanying notes 126–29. For both full liability and partial liability rules, to show the non-existence of an equilibrium in pure strategies, one must also show, as one can, that there is no asymmetric equilibrium in which one actor meets the standard of care and the other does not.