Sovereign Debt and Moral Hazard: The Role of Collective Action and Contractual Ambiguity

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Abstract

The ambiguous phrasing of pari passu (equal treatment) clauses in sovereign debt contracts has long baffled commentators. We show that in the presence of asymmetric information on a sovereign borrower’s ability to pay, an ambiguous pari passu clause gives rise to a collective action problem among creditors that can reduce sovereign moral hazard. By varying the clause ambiguity, parties can induce an (ex ante) optimal probability of costly renegotiation breakdown resulting from creditors’ failure to coordinate. As information asymmetry decreases, a pari passu clause becomes a coarser instrument for configuring creditors’ incentives and thereby resolving moral hazard.

Keywords: Sovereign debt, pari passu clauses, strategic bargaining

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“... so that the door might not be shut in the face of borrowers.”

Babylonian Talmud

1 Introduction

The pari passu clause is one of the most common, most commented upon, and most controversial clauses in sovereign debt contracts. The clause, a version of which is included in the vast majority of unsecured sovereign bonds (Gulati and Scott, 2013; p. 187), provides that the bonds, and/or the sovereign’s payment obligations, shall rank equally or pari passu with other unsecured debt of the sovereign. According to some interpretations, the clause prohibits a sovereign borrower from selectively paying one group of creditors and not paying others. Recently, the pari passu clause has drawn considerable public attention in the wake of high-stakes litigation brought by creditors against the Republic of Argentina.1

Although corporate debt agreements occasionally include similar provisions, the pari passu clause has special significance in sovereign bond contracts. Payment obligations of sovereign states are notoriously difficult to enforce: sovereigns cannot be forced into bankruptcy and often hold few non-domestic assets that creditors can attach. Creditors may thus face difficulties collecting their debt even if their right to payment is clear. (Bulow and Rogoff, 1989a).

When a sovereign debtor is unable to pay its creditors, it often proposes to exchange its outstanding debt for new debt with less onerous payment terms (such as a lower principal amount or interest rate). Whereas some creditors fearing an imminent default may agree to reduce their debt (“Consenting Creditors”), other creditors may hold out by retaining their original bonds (“Holdout Creditors”). In this restructuring context, a sovereign debtor might threaten to pay nothing to Holdout Creditors to induce other creditors to consent to reduce their debt.2

Arguably, the purpose of the pari passu clause is to prohibit such discrimination between Consenting and Holdout Creditors. If the pari passu clause prohibited discrimination, sovereign debtors would be barred from paying Consenting Creditors their renegotiated (reduced) claims without also paying Holdout Creditors their original (full) claims.3 If creditors’ claims had to be treated equally, therefore, more creditors may hold out in the hope of recovering their entire debt and it may become more difficult for distressed countries to effect a restructuring.

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2Older sovereign bonds limited the power of a majority of creditors to bind dissenting creditors. Some of the more recent sovereign bonds include “collective action clauses” that empower a supermajority of creditors to bind all creditors, but require such a supermajority in any bond issue. Because Holdout Creditors can often accumulate a blocking minority in one or more bond issues, collective action clauses do not fully resolve the holdout problem. See Weidemaier, 2013; Buchheit et al., 2013.
3The potency of a pari passu clause to prohibit creditor discrimination (when it does) lies in its enforceability. A sovereign debtor that wishes to pay Consenting Creditors in violation of a pari passu clause must often process payments through financial intermediaries like banks and trustees. Holdout Creditors can enforce a pari passu clause by enjoining such intermediaries from processing payments to Consenting Creditors.
Whether the pari passu clause in fact prohibits such discrimination is subject to substantial controversy. Many commentators have argued, based on historical and policy considerations, that the clause merely prohibits sovereigns from creating a class of creditors that is legally senior (e.g., Buchheit and Pam, 2004; Gulati and Klee, 2001). Others have presented arguments that the clause should also apply to de facto discrimination among creditors (Semkow, 1984; Bratton, 2004). Most commentators, including many who have taken a positive or normative position on the interpretation of the clause, concede that its meaning is not clear. For example, Buchheit and Pam (2004, p. 871) described the clause as possessing a “measure of opacity”; Montelore (2013) described it as “obscure”; and Weidemaier, Scott, and Gulati (2011) remarked that “it is fair to say that no one really knows what the pari passu clause means, something that even eminent practitioners have long acknowledged.” Moreover, several commentators puzzled about the persistence of ambiguously-phrased clauses and the failure of contracting parties to clarify them despite ongoing disputes over their scope and meaning (e.g., Gulati and Scott, 2013; Goss, 2014; Buchheit and Martos, 2014).

As if to compound the confusion, the pari passu clause comes in at least three different formulations, each entailing a different likelihood of being interpreted “broadly” to prohibit de facto discrimination (Gulati and Scott, 2013, p. 187; Weidemaier, 2013). Some clauses specify only that the bonds must rank equally or pari passu with other debt; other clauses specifically provide that the payment obligations rank equally or pari passu with other debt; and a third version further requires that the bonds be paid in accordance with their equal or pari passu ranking.

Two court rulings that sided with holdout creditors—one by a Court of Appeals in Belgium in 2000 and another by the Second Circuit Court of Appeals in New York in 2012—dispelled the notion, professed by some commentators, that the clause is universally understood to permit de facto discrimination. Yet because they involved idiosyncratic circumstances, these rulings provide only limited guidance for the resolution of future disputes (see, e.g., Gulati and Klee, 2001; Alfaro, 2015; Ku, 2014).

In this paper, we present a three-period model of sovereign debt that sheds light on the persistent ambiguity of pari passu clauses. We consider a sovereign (Country) and a continuum of creditors (Creditors) that negotiate in the first period a loan amount and a probability that a pari passu clause would be interpreted to prohibit de facto discrimination. In the second period, Country must choose a policy from a set of policies, where each policy entails a different probability of failure and a riskier policy yields a higher payout upon success. In the third period, the payout of Country’s chosen policy is realized. If the policy succeeds, its payout is sufficient to pay off Creditor’s debt. If the policy fails, by contrast, the policy’s realized payout falls short of Country’s debt. The realized payout upon failure is private information to Country; Creditors only know the payout distribution. Restructuring negotiations under asymmetric information then follow in the shadow of the probability that the pari passu clause would be interpreted

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4The 2000 decision was preliminary, rendered on an ex parte motion, and involved a foreign court interpreting New York Law. The 2012 decision was based, in part, on equitable considerations and on Argentina’s enactment of the Lock Law which may have violated even a narrow interpretation of the pari passu clause. The Second Circuit Court of Appeals specifically noted that it did not decide whether “any non-payment that is coupled with payment on other debt” would breach the pari passu clause. See NML Capital. 246 F.3rd at 264, fn. 16; see also White Hawthorne LLC v. Republic of Argentina, 2016 WL 7441699 (refusing to prohibit de facto discrimination against certain holdout creditors).
broadly.

This setup is intended to capture the effect of restructuring negotiations on Country’s choice of policy and thereby on the optimal design of a loan agreement. In particular, the first-period loan negotiations are conducted in the shadow of Country’s anticipated choice of policy in the second period, which is shaped by the expected outcome of the restructuring negotiations in the final period. In the presence of weak contractual enforcement of Country’s payment obligations, the design of restructuring negotiations through a *pari passu* clause is a key instrument for curbing Country’s moral hazard.

To model the equilibrium outcome of the restructuring negotiations following a policy failure, we assume that a sufficiently large fraction of Creditors can make Country a take-it-or-leave-it restructuring demand requiring that Country pays them a reduced debt amount. We call the fraction of Consenting Creditors out of all creditors the participation rate. The complementary fraction of (Holdout) Creditors retains their original debt. Country cannot pay Creditors more than its realized policy payout and, if the *pari passu* clause is interpreted broadly, may not pay Consenting Creditors their restructured debt without paying Holdout Creditors their debt in full.

If Country fails to pay off Creditors according to the realized interpretation of the *pari passu* clause, Country defaults and incurs default costs and Creditors receive nothing. More specifically, Country can avoid incurring default costs by (i) paying Consenting Creditors their demand and paying Holdout Creditors nothing if the *pari passu* clause is interpreted narrowly or (ii) paying Consenting Creditors their demand and paying Holdout Creditors their claim in full if the *pari passu* clause is interpreted broadly.

We solve the game using the notion of restructuring equilibrium. For a given probability that the *pari passu* clause would be interpreted broadly and a given level of asymmetric information between Country and Creditors, a restructuring equilibrium consists of a participation rate and a restructuring demand that satisfy two conditions: (i) Consenting Creditors’ restructuring demand maximizes their expected recovery given the participation rate; and (ii) no creditor has incentives to deviate from his equilibrium strategy (consent or hold out) given the participation rate and Consenting Creditors’ restructuring demand.

Our model brings to the fore the significance of uncertainty embedded in the *pari passu* clause for the outcome of restructuring negotiations. The more likely a *pari passu* clause is to be interpreted broadly, the lower the equilibrium participation rate and the higher the associated probability of Country’s default. By varying the probability that a *pari passu* clause would be interpreted broadly, Country and Creditors can therefore affect the ex ante probability of costly default in the event of a policy failure. A higher probability of costly default given a policy failure in turn induces Country to choose a safer policy. A *pari passu* clause improves Country’s welfare if the increase in the policy payout outweighs the corresponding increase in expected default costs. Moreover, a *pari passu* clause that increases Country’s welfare also allows Country to borrow more from Creditors.

To see how a *pari passu* clause shapes Creditors’ incentives during restructuring negotiations, observe that a stronger clause (i.e., one that is more likely to be interpreted broadly) produces stronger incentives to hold out. As more creditors hold out and the participation rate decreases, Consenting Creditors lower their restructuring demand. In
equilibrium, the decrease in Consenting Creditors’ restructuring demand is lower than the corresponding increase in Holdout Creditors’ claim. A stronger clause therefore results in a higher aggregate claim by Consenting and Holdout Creditors given a broad interpretation of the clause. Thus, as the *pari passu* clause becomes stronger, Country’s probability of default given a policy failure increases both because a broad interpretation of the clause is more likely and because Country is less likely to meet its payment obligations to Consenting and Holdout Creditors when the clause is interpreted broadly.

Asymmetric information is essential for the *pari passu* clause to produce this dynamic. If the degree of information asymmetry is sufficiently low, Consenting Creditors’ optimal restructuring demand is one where Country either never defaults irrespective of the interpretation of the clause or always defaults given a broad interpretation of the clause (depending on the participation rate). An equilibrium in which Country never defaults is of no avail because it does not curb Country’s moral hazard. On the other hand, a demand that induces certain default cannot be sustained in equilibrium for it would leave Holdout Creditors with no recovery. It consequently takes a sufficiently high degree of information asymmetry for the *pari passu* clause to give rise to equilibria involving an interior probability of Country’s default.

More generally, a *pari passu* clause is a renegotiation-proof stochastic mechanism for producing limited holdout incentives, which strengthens creditors’ bargaining position against their (ex post) collective interest. The benefit of perturbing the restructuring process stems from the fact that renegotiation distorts sovereigns’ interim (non-contractible) investment decisions. By increasing the probability of a costly default upon a failure to pay creditors’ debt, a *pari passu* clause aligns sovereigns’ interim investment incentives. The optimal strength of a *pari passu* clause accordingly depends on the benefits from reducing sovereign moral hazard versus the associated default costs.⁵

**Prior Literature:**

Our model builds on the extensive literature on sovereign debt. One branch of this literature revolves around mechanisms for ensuring sovereign repayment that substitute for direct legal enforcement. In a seminal paper, Eaton and Gersovitz (1981) present a model of sovereign debt with post-default penalties that provide repayment incentives even when the underlying payment obligations cannot be directly enforced. Subsequent papers have proposed different theories on the sources of default penalties (see Grossman and Van Huyck, 1988; Bulow and Rogoff (1989b); Fernandez and Rosenthal, 1990; Cole and Kehoe, 1995; Eaton, 1996; Panizza et al., 2009, Chabot and Santarosa, 2017).

Bulow and Rogoff (1989b) incorporate the possibility of renegotiations (without holdouts) into a model of sovereign lending. Fernandez and Fernandez (2007) consider the case in which debt can be renegotiated and countries may engage in strategic default. Atkeson (1991), Schwartz and Zurita (1992), and Boot and Kanatas (1995) present models of sovereign borrowing that include sovereign moral hazard.⁶

⁵Other papers have suggested that credit default swaps play a similar role both in private and sovereign debt contracts (Bolton and Oehmke, 2011; Sambalaibat, 2012).

⁶In addition to the classic moral hazard problem relating to a debtor’s choice of policy that affects the probability of distress, other related forms of moral hazard include a debtor’s decision to seek a restructuring and creditors’ decision to lend given the possibility of bailout by international institutions. Buchheit et al., 2013.
Buchheit and Gulati (2000) and Schwarz (2000), among others, have examined the holdout problem and the associated difficulties in renegotiating sovereign debt. Bolton and Jeanne (2007) present a model where competition among lenders can result in sovereign debt that is excessively difficult to renegotiate. Ghosal and Thampanishvon (2013) argue that strengthening collective action away from unanimity can reduce the holdout problem but aggravate moral hazard.

Most of the literature on pari passu clauses has centered on the interpretation of the clause (Semkow, 1984; Wood, 1995; Gulati and Klee, 2001; Bratton, 2004; Buchheit and Pam, 2004), on discussions of the case law (Gulati and Klee, 2001; Montelore, 2013; Weidemaier, 2013; Ku, 2014; Alfaro, 2015; Tsang, 2015), and on an empirical investigation of the prevalence of various versions of the clause (Weidemaier, Scott and Gulati, 2011; Gulati and Scott, 2013). Prior work has noted that a broad reading of the pari passu clause that prohibits de facto discrimination can inhibit consensual restructurings (Gulati and Klee, 2001) and may reduce sovereign moral hazard (Bratton, 2004), but has not provided a formal account of these effects.

More broadly, our paper is related to the literature on the design of incentive-compatible mechanisms. A general problem of designing such mechanisms is the distorting effect of ex post renegotiation on ex ante incentives (Aghion, Dewatripont, and Rey, 1994; Evans, 2012; Neeman and Pavlov, 2013). This paper suggests that the pari passu clause is a potentially widely-used mechanism that exploits a collective action problem among creditors to inhibit ex post renegotiation of a sovereign debtor’s obligations. It thereby contributes to the literature on contractual incompleteness, which has typically attributed vagueness or unspecifity of contract terms to transaction costs or bounded rationality (see, e.g., Hart and Moore, 1999). In our model, contractual ambiguity is efficiency-enhancing and strategic, as in Spier (1992) and Bernheim and Whinston (1998).

The paper is organized as follows. Section 2 sets up the model and introduces sovereigns moral hazard. Section 3 presents and classify restructuring equilibria and Section 4 derives the equilibrium outcomes. Section 5 characterizes the optimal strength of a pari passu clause. Section 6 considers three extensions. Section 7 concludes.

2 Model

2.1 Setup

We consider a three-period game between a country (Country) and a unit mass of identical competitive creditors (Creditors). Both Country and Creditors are risk-neutral and the market interest rate is zero.

In period 0, Creditors lend Country \( k \leq 1 \) in exchange of Country’s commitment to repay Creditors 1 in period 2. As will be clear soon, Country cannot commit to pay back more than 1. The loan amount, \( k \), is determined such that Creditors break even in expectation. The loan includes a pari passu clause (Clause) along with a probability \( w \in [0, 1] \) that the Clause would be interpreted “broadly” to prohibit de facto discrimination. If the Clause is interpreted broadly, then in period 2 Country may not pay some Creditors but
not others.

In period 1, Country must choose a policy \( p \in (0, 1] \). We think of a policy as any fiscal or monetary measure designed to produce a long-term fiscal payout such as tax, pension, or currency reform. A policy \( p \) succeeds with probability \( p \) and fails with the complementary probability. If a policy \( p \) succeeds it yields a payout of \( r(p) \), where \( r(1) > 1 \), \( r'(p) < 0 \), and \( r''(p) \leq 0 \). A lower \( p \) accordingly represents a policy that yields a higher payout if it succeeds but is more likely to fail. If the policy fails, its payout (independent of \( p \)) is a uniform random variable with mean \( \mu = 3/4 \) and Country-specific support \([s, \bar{s}]\), where \( s = \bar{s} - s \in (0, 1/2] \).

In period 2, the observable outcome - success or failure - of Country’s chosen policy is realized. The realized value of the policy payout, however, is private information to Country. If Country pays Creditors 1, the game ends. If Country does not pay Creditors, a restructuring phase follows, in which Creditors and Country negotiate a reduction of Country’s debt. Country cannot pay Creditors more than its policy payout. If Country and Creditors fail to reach a restructuring agreement, Country incurs default costs of 1 and Creditors obtain 0.\(^7\)

This setup is designed to capture two essential features of sovereign debt. The first is the non-enforceability of Country’s policy choice, a choice captured by the probability \( p \) in our model. Country’s choice of policy in turn affects both Country’s payout and probability of default. The second essential feature of sovereign debt our model captures is the asymmetric information underlying restructuring negotiations, whose magnitude is represented by the support \( s \) of Country’s failed policy payout. This information asymmetry can be interpreted as a measure of economic and political transparency. The more transparent Country is, the less uncertainty Creditors face during restructuring negotiations.\(^8\)

### 2.2 The restructuring phase

The restructuring phase proceeds in two stages. In stage 1, a mass \( \alpha \in [\alpha_{\min}, 1] \) of Creditors, called Consenting Creditors, may make Country a take-it-or-leave-it demand of \( d \in (0, \alpha] \), where \( \alpha \) represents Creditors’ participation rate (i.e., the fraction of Consenting Creditors out of all Creditors) and \( \alpha_{\min} \) stands for Creditors’ participation constraint. The participation constraint is the minimum fraction of Creditors that must consent and be paid according to a restructuring plan for Country to avoid the (political and economic) costs from defaulting on its payments to Creditors. If Creditors do not make a demand, Country has to pay Creditors in full to avoid default costs. If Creditors make a demand, the non-consenting creditors, called Holdout Creditors, retain their original debt with an aggregate claim of 1.

In stage 2, the interpretation of the Clause is realized. If the Clause is interpreted narrowly, Country may avoid default costs by paying \( d \) to Consenting Creditors, without

\[^7\]We assume that Country would rather pay Creditors if it is indifferent between incurring default costs and paying Creditors.

\[^8\]An alternative interpretation of the realized payout of a failed policy is the maximum amount that Country’s government is politically willing to pay Creditors rather than default, which amount is lower in the event of a policy failure than a policy success.
If the Clause is interpreted broadly, by contrast, Country may not discriminate between Consenting and Holdout Creditors; to avoid default costs, Country must pay $d$ to Consenting Creditors and $1$ to Holdout Creditors.

If the policy succeeds, the realized policy payout is greater than Country’s debt of $1$. Because Country’s default costs are equal to its debt, Country will pay off Creditors in full (whether or not Creditors make a demand to Country). If the policy fails, by contrast, Country will default if (i) its realized policy payout is less than $d$ irrespective of the realized interpretation of the Clause; or (ii) its realized policy payout is less than $d + 1 - \alpha$ and the Clause is interpreted broadly.

Our modeling of the restructuring negotiations is intended to capture the fact that Country can avoid (or reduce) the penalty associated with a sovereign default if it reaches and implements a restructuring plan approved by a significant fraction of creditors ($\geq \alpha_{\text{min}}$). A broad interpretation of the Clause, however, prevents the implementation of a restructuring unless hold-out creditors are paid as well.

### 2.3 Moral hazard

As a benchmark, consider the outcome of the restructuring negotiations in period 2 given that the Clause is certain to be interpreted narrowly ($w = 0$). Clearly, all Creditors would rather consent to a restructuring plan than hold out for holding out yields no recovery. Because Country always accepts a demand $d < \underline{s}$ and always rejects a demand $d \geq \overline{s}$, Creditors choose a demand $d \in [\underline{s}, \overline{s})$ to maximize their expected recovery of

$$d \times P_{\text{nd}}(d, s),$$

where $P_{\text{nd}}(d, s) = (\overline{s} - d)/s \in (0, 1]$ is the probability that Country does not default and therefore can meet Creditors’ demand. Because Creditors’ expected recovery decreases with $d$ for $d \in (\underline{s}, \overline{s})$, Creditors’ optimal demand is $\underline{s}$.\textsuperscript{9} The information asymmetry between Country and Creditors thus causes Creditors to make a cautious restructuring demand, which ensures that Country never defaults.

Consider next Country’s choice of policy in period 1. Let $R(p) \equiv pr(p) + (1 - p)\mu$ denote

\textsuperscript{9} The derivative of (1) with respect to $d$ is $(\overline{s} - 2d)/s < 0$ for $d \in (\underline{s}, \overline{s})$ (because $\underline{s} \geq \overline{s}/2$).
policy $p$’s expected payout, where $r(p)$ and $\mu$ are the expected payout in case of success and failure, respectively. Country’s optimal policy maximizes its interim (period 1) payoff of $R(p) - T(p)$, where $T(p)$ is Country’s total payments in period 2. Country thus equates the marginal payout of a safer policy and the corresponding marginal cost: $R'(p) = T'(p)$. By contrast, the socially optimal (interior) policy satisfies $R'(p) = 0$ so that the marginal payout of a safer policy is nil.

Now, given that the Clause is certain to be interpreted narrowly, $T(p) = p + (1 - p)s$, because Country pays Creditors 1 when the policy succeeds and $s$ when the policy fails. Because $T'(p) = 1 - s > 0$, Country’s total payments in period 2 increase with $p$, the probability the policy succeeds. Country will therefore choose a riskier policy than the socially optimal one, thereby trading off a higher expected policy payout for lower expected payments to Creditors. Of course, Country is consequently worse off because Creditors recover the loan amount (on expectation) so Country alone bears the costs of its suboptimal choice of policy. Relative to the socially optimal policy, Country’s privately optimal policy produces a lower payout and involves a lower loan amount Creditors would be willing to lend Country.\footnote{We assume that Country benefits from a higher loan amount and cannot commit to a borrowing cap. As a result, Country cannot reduce moral hazard by promising to pay back Creditors less than 1 (and borrowing less).}

\section{Restructuring Equilibria}

In this section, we turn to the case in which the probability the Clause is interpreted broadly is strictly positive. We first define and classify restructuring equilibria and then flesh out the equilibrium restructuring demand and participation rate.

\textbf{Definition 1 (restructuring equilibrium)} A restructuring equilibrium is a pair of a participation rate ($\alpha$) and a restructuring demand ($d$) such that:

(i) The restructuring demand maximizes Consenting Creditors’ recovery given the participation rate;

(ii) No creditor can profitably switch position (consent or hold out) given the restructuring demand and the participation rate.

Two types of equilibria emerge in our setup: a full-participation equilibrium, in which all Creditors participate ($\alpha = 1$) and make a restructuring demand of $s$; and a partial-participation equilibrium, in which some Creditors participate and others hold out ($\alpha < 1$) and Consenting Creditors make a restructuring demand equal to or lower than $s$. A partial participation equilibrium is unconstrained if Creditors’ participation constraint is not binding and is constrained if this constraint is binding. To sustain a full-participation equilibrium, no mass of Creditors could profitably deviate to holding out; to support an unconstrained partial-participation equilibrium, Consenting Creditors and Holdout Creditors must obtain the same recovery rate; and to support a constrained equilibrium wherein the participation rate is equal to $\alpha_{\text{min}}$ (the participation constraint), Consenting Creditors’ recovery rate may not exceed Holdout Creditors’ recovery rate.
In the rest of this section, we formally represent the two-prong equilibrium condition. To this end, consider first the probability that Country does not default given that the Clause is interpreted broadly as a function of Consenting Creditors’ demand, the participation rate, and the degree of information asymmetry between Country and Creditors:

\[
P_{nd}(\alpha, d, s) \begin{cases} 
\quad = 1 & \text{if } d + (1 - \alpha) \in [0, \underline{s}]; \\
\quad = 0 & \text{if } d + (1 - \alpha) \in [\underline{s}, \overline{s}) \cup [\overline{s}, 1]. 
\end{cases}
\] (2)

If the sum of Creditors’ demands, \(d + (1 - \alpha)\), is less than \(\underline{s}\), the lower bound of a failed policy payout, Country never defaults (first line). If the sum of Creditors’ demands is greater than \(\overline{s}\), the upper bound of a failed policy payout, Country defaults with certainty if the Clause is interpreted broadly (third line). Finally, if the sum of Creditors’ demands is strictly between \(\underline{s}\) and \(\overline{s}\), Country defaults with a probability strictly between 0 and 1 if the Clause is interpreted broadly (middle line). \(^{11}\)

Consider now Consenting Creditors’ optimal restructuring demand as a function of the participation rate (first equilibrium condition). Any restructuring demand cannot exceed the participation rate, because Consenting Creditor cannot demand more than their claim of \(\alpha\). Moreover, Consenting Creditors’ optimal demand for any \(w > 0\) is no greater than \(\underline{s}\) because \(\underline{s}\) is Consenting Creditors’ optimal demand for \(w = 0\) and because Consenting Creditors’ optimal demand (weakly) decreases with \(w\). Given a participation rate \(\alpha\), Consenting Creditors’ optimal restructuring demand solves

\[
\max_{d \in [0, \min(\alpha, \underline{s})]} d \times [(1 - w) + wP_{nd}(\alpha, d, s)].
\] (3)

An optimal restructuring demand maximizes Consenting Creditors’ expected recovery: the restructuring demand multiplied by the probability that Country meets the demand. More specifically, the first term in the square brackets, \(1 - w\), is the probability that the Clause is interpreted narrowly, in which case Country always meets Consenting Creditors’ demand. The second term, \(wP_{nd}\), is the joint probability that the Clause is interpreted broadly and that Country can meet both Consenting Creditors’ and Holdout Creditors’ demands.

The next definition sorts Consenting Creditors’ demands.

**Definition 2 (classification of restructuring demand)** For a given participation rate \((\alpha)\) and a given level of asymmetric information between Country and Creditors \((s)\), Consenting Creditors’ demand \((d)\) is: (i) a “default demand” if \(P_{nd}(\alpha, d, s) = 0\); (ii) a “no-default demand” if \(P_{nd}(\alpha, d, s) = 1\); or (iii) an “interior demand” if \(P_{nd}(\alpha, d, s) \in (0, 1)\).

Consenting Creditors’ demand is a default demand if Country always defaults given a broad interpretation of the Clause; a no-default demand if Country never defaults irrespective of the interpretation of the Clause; and an interior demand if Country’s

\(^{11}\)We derive the expression for \(P_{nd}(\alpha, d, s)\) for this case in the Appendix.
probability of default given a broad interpretation of the Clause is strictly between 0 and 1. Consenting Creditors’ optimal demand (although not necessarily an equilibrium demand) is either a default demand of \( \min\{\alpha, s\} \), a no-default demand of \( s - (1 - \alpha) \), or an interior demand which is equal to or lower than \( s \).

Turning to the second equilibrium prong, no creditor would have an incentive to switch position under an unconstrained partial-participation equilibrium iff

\[
(d/\alpha) \times [1 - w + wP_{nd}(\alpha, d, s)] = wP_{nd}(\alpha, d, s). \tag{4}
\]

The left-hand side is Consenting Creditors’ recovery rate: the ratio of the restructuring demand and the participation rate multiplied by the probability that Country meets Consenting Creditors’ demand. The right-hand side is Holdout Creditors’ recovery rate: the joint probability that the Clause is interpreted broadly and that Country does not default (recall that Holdout Creditors’ demand consists of their entire claim). In any equal recovery (unconstrained) equilibrium, Consenting Creditors’ restructuring demand and the participation rate must satisfy both (3) and (4).

4 Equilibrium Outcomes

We begin by considering a full-participation equilibrium in which all Creditors participate in a restructuring plan. In this equilibrium, given the distribution of Country’s failed policy payout, Creditors make a riskless restructuring demand equal to the lower bound of Country’s policy payout \( s \), which Country always accepts. Country thus never defaults and its payment to Creditors in case of a policy failure is \( s \). The next proposition presents a necessary and sufficient existence condition for a full-participation equilibrium as a function of \( w \) and \( s \).

**Proposition 1 (full-participation equilibrium)** There exists a payoff dominant full-participation equilibrium iff \( w \in [0, s] \) (Area A in Figure 1).

**PROOF.** See the Appendix.

To see the intuition for the existence of a full-participation equilibrium for \( w \in [0, s] \), observe that if all Creditors participate, their optimal restructuring demand is \( s \). Because Country always accepts Creditors’ demand, Creditors’ recovery rate is \( s \). By contrast, holding out yields a recovery rate no greater than \( w \), the probability that the Clause will be accorded a broad interpretation. Because holding out yields a lower recovery rate than consenting, no creditor has incentives to deviate to holding out.

A full-participation equilibrium is payoff dominant because under any partial-participation equilibrium Holdout Creditors’ recovery rate is capped at \( w \) and Consenting Creditors’ recovery rate cannot be higher than Holdout Creditors’ recovery rate. Both Consenting and Holdout Creditors thus obtain a higher recovery rate under a full-participation
equilibrium (where \( w \in [0, \underline{s}] \)) than under a partial-participation equilibrium.\(^\text{12}\)

There does not exist a full-participation equilibrium for \( w \in (\underline{s}, 1] \) because, given that all other Creditors participate and make a restructuring demand of \( \underline{s} \), an infinitesimal mass of consenting creditors can profitably deviate to holding out. In the limit, as the deviating mass approaches zero, Country’s probability of not defaulting conditional on a broad interpretation of the Clause tends to 1. The deviating creditors’ recovery rate would consequently be arbitrarily close to \( w \) and thus higher than their (putative) equilibrium recovery rate of \( \underline{s} \). These holdout incentives in turn disrupt a full-participation equilibrium.

We now turn to partial-participation equilibria in which some creditors participate in a restructuring plan while other creditors hold out. In any such equilibria, Consenting and Holdout Creditors obtain equal recovery rates; we shall accordingly call these equilibria “equal recovery equilibria.” We shall assume throughout most of the analysis that the participation constraint is not binding and will comment later on the implications of relaxing this assumption. The next proposition presents necessary and sufficient existence conditions for equal recovery equilibria for \( w \in (\underline{s}, 1] \) as well as a key property underlying them.

**Proposition 2 (equal recovery equilibria)** Let \( \bar{w} \equiv \max\{\underline{s}, s/(1 - \underline{s})\} \) (see Figure 1) and assume that the participation constraint is not binding.\(^\text{13}\)

(a) For \( w \in (\underline{s}, \bar{w}] \) (Area B in Figure 1) there exists a unique partial participation equilibrium in which (i) Consenting Creditors make an interior demand and (ii) Consenting and Holdout Creditors obtain equal recovery rates.

(b) For \( w \in (\bar{w}, 1] \) (Area C in Figure 1) there does not exist an equal recovery equilibrium.

**PROOF.** See the Appendix, which includes a full characterization of the equilibrium outcomes.

We begin by explaining why for \( w \in (\underline{s}, 1] \) any candidate equal recovery equilibrium must involve an interior restructuring demand; i.e., a demand under which Country’s probability of default given a broad interpretation of the Clause is strictly between 0 and 1.

A default demand cannot be part of an equilibrium for any \( w \). This is because under any putative equilibrium involving a default demand, Consenting Creditors’ recovery rate is strictly positive but Holdout Creditors’ recovery rate is nil. Holdout creditors can consequently profitably deviate to consenting, thereby upsetting the putative equilibrium.

On the other hand, a no-default demand cannot be part of an equilibrium for any \( w \in (\underline{s}, 1] \). To see why, observe that Holdout Creditors’ recovery rate under a no-default demand is \( w \) (the probability that the Clause would be interpreted broadly).

\(^\text{12}\)As we show in Proposition 1A in the Online Appendix, for \( w \in [\bar{w}(s), \underline{s}] \) there exists a partial-participation equilibrium in which Consenting Creditors make a no-default demand and both Consenting and Holdout Creditors’ equilibrium recovery rate is \( w \).

\(^\text{13}\)We suppress the argument \( s \) of \( \bar{w} \).
If a no-default demand were part of an equal recovery equilibrium, Consenting Creditors’ recovery rate as well as Creditors’ recovery would have to be \( w \) too, which is greater than Creditors’ full-participation recovery of \( s \) for any \( w \in (s, 1] \). But under a no-default demand Creditors’ recovery is strictly less than \( s \) given a narrow interpretation of the Clause and \( s \) given a broad interpretation (because the sum Creditors’ demands is \( s \)), implying that Creditors’ recovery under a no-default demand must be strictly less than \( s \). Thus, if Consenting Creditors made a no-default demand for any \( w \in (s, 1] \), Holdout Creditors would obtain a higher recovery rate than Consenting Creditors, providing the latter incentives to switch to holding out. An equilibrium for \( w \in (s, 1] \) therefore exists for if and only if Consenting Creditors’ optimal restructuring demand is neither a default demand nor a no-default demand but rather an interior demand.  

There does not exist an equilibrium for any pair \((w, s)\) in Area C in Figure 1 because in this region there does not exist an interior demand that maximizes Consenting Creditors’ expected recovery and satisfies the equal recovery condition for any participation rate. In particular, for high participation rates Consenting Creditors’ optimal demand is either a no-default or an interior demand involving unequal recovery rates, which gives consenting creditors incentives to hold out. The participation rate consequently drops until Consenting Creditors’ optimal demand becomes a default demand. A default demand, conversely, gives holdout creditors incentives to consent and thereby raise the participation rate. The interchanging incentives to hold out (for high participation rates) or consent (for low participation rates) thus frustrate any potential equal-recovery equilibrium.  

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14 More generally, a full-participation equilibrium maximizes Creditors’ recovery for any \( w \). Under a partial-participation equilibrium, Creditors maximize their expected recovery given a broad interpretation of the Clause by having Consenting Creditors make a no-default demand. Because a no-default demand is strictly less than \( s \), however, it fails to maximize Creditors’ recovery given a narrow interpretation of the Clause.

15 If an equilibrium does not exist, Country defaults with certainty.

16 As we show in the proof of Proposition 2, an equal recovery equilibrium exists for values of \( w \) and \( s \) for which \( d^*(w, s) \geq \alpha^*(w, s)/2 \) independently of the distribution of Country’s failed policy payout.
For any pair \((w, s)\) in Area B in Figure 1 there is a unique equal-recovery equilibrium because Holdout Creditors' recovery rate is higher than Consenting Creditors' recovery rate for \(\alpha\) sufficiently close to 1 and because (as we shortly explain) Holdout Creditors' recovery rate decreases more rapidly as the participation rate decreases than Consenting Creditors' recovery rate, given that Consenting Creditors' restructuring demand is optimally adjusting to the lower participation rate. These two facts together imply that as the participation rate decreases, Consenting and Holdout Creditors' recovery rates cross no more than once.

A drop in the participation rate has a greater (negative) effect on Holdout Creditors' recovery rate than on Consenting Creditors' recovery rate because Consenting Creditors' optimal restructuring demand, \(\hat{d}(\alpha)\), decreases by less than the corresponding drop in \(\alpha\) (as we show in the proof of Proposition 2). As a result, the ratio \(\hat{d}(\alpha)/\alpha\) increases as \(\alpha\) decreases. Thus, given a broad interpretation of the Clause, Consenting Creditors' recovery rate \((\hat{d}(\alpha)/\alpha \times P_{nd})\) decreases less steeply as the participation rate decreases than Holdout Creditors' recovery rate \((P_{nd})\). Because Consenting Creditors' recovery rate given a narrow interpretation of the clause \((\hat{d}(\alpha)/\alpha)\) increases as the participation rate decreases, Consenting Creditors' overall recovery rate is less responsive to a drop in the participation rate than Holdout Creditors' recovery rate. Finally, Creditors' participation constraint would suppress an equal recovery equilibrium if it were greater than the equilibrium participation rate \((\alpha_{\text{min}} > \alpha^*)\). In turn, the participation constraint can give rise to new equilibria in which the participation rate is equal to the participation constraint. Such equilibria exist if and only if, given Consenting Creditors' optimal demand, Consenting Creditors' recovery rate does not exceed Holdout Creditors' recovery rate.

The next proposition considers the effect of an increase in \(w\) on the equilibrium quantities.

**Proposition 3 (effects of a stronger clause)** In an equal recovery equilibrium, both the participation rate and Consenting Creditors' demand (weakly) decrease with \(w\), but the sum of Creditors' demands increases with \(w\). Consequently, given that Country's policy fails, (i) Country's probability of default increases with \(w\), and (ii) the sum of Country's payments to Creditors and default costs increases with (and is equal to) \(w\).

The effect of an increase in \(w\) on the equilibrium participation rate and restructuring demand under an equal recovery equilibrium follows from the corresponding effect on Consenting Creditors' and Holdout Creditors' recovery rates. Other things being equal, an increase in \(w\) increases Holdout Creditors' recovery rate and decreases Consenting Creditors' recovery rate. More creditors consequently hold out, thereby lowering the participation rate. The drop in the participation rate in turn (weakly) lowers Consenting Creditors' optimal demand, which decreases the participation rate further and so on. The resulting equilibrium accordingly involves a lower participation rate and a (weakly) lower Consenting Creditors' demand. Furthermore, as we explain below, the sum of Creditors' equilibrium demands, \(d^* + (1 - \alpha^*)\), increases with \(w\) and therefore Country's equilibrium probability of default increases with \(w\) as well.

To see why \(d^* + (1 - \alpha^*)\) increases with \(w\), suppose that this sum remained unchanged or decreased as \(w\) increases. Since an increase in \(w\) lowers the participation rate, for
the sum of Creditors’ equilibrium demands to remain unchanged or to decrease with $w$, Consenting Creditors’ equilibrium demand must decrease by as much or more than the corresponding drop in the equilibrium participation rate.

If Consenting Creditors’ equilibrium demand declined by as much or more than the drop in the equilibrium participation rate, however, Country’s equilibrium probability of no-default conditional on a broad interpretation of the Clause would either not change or increase with $w$ (i.e., $dP_{nd}(\alpha^*,d^*)/dw \geq 0$). Holdout Creditors’ recovery rate given a broad interpretation of the Clause, $P_{nd}(\alpha^*,d^*)$, would consequently increase with $w$. By contrast, Consenting Creditors’ recovery rate given a narrow interpretation of the Clause $(d^*/\alpha^*)$ would decrease with $w$ and their recovery rate given a broad interpretation of the Clause $(d^*/\alpha^* \times P_{nd}(\alpha^*,d^*))$ would either decrease or increase with $w$ (depending on whether the decrease in $d^*/\alpha^*$ were less or greater than the increase in $P_{nd}(\alpha^*,d^*)$). But even if Consenting Creditors’ recovery rate given a broad interpretation of the Clause increases with $w$, it could not increase by more than the corresponding increase in Holdout Creditors’ recovery rate, because the driving force for Consenting Creditors’ higher recovery rate—a higher probability of Country not defaulting—affects Holdout Creditors’ recovery rate more than Consenting Creditors’ recovery rate. Holdout Creditors’ recovery rate would consequently be higher than Consenting Creditors’ recovery rate, in violation of the equilibrium condition of equal recovery rates.

It follows that for Consenting and Holdout Creditors to obtain equal recovery rates as $w$ increases, the decline in Consenting Creditors’ equilibrium demand must be smaller than the drop in the equilibrium participation rate. But if Consenting Creditors’ equilibrium demand decreases by less than the drop in the equilibrium participation rate, the sum of Creditors’ equilibrium demands $(d^* + (1 - \alpha^*))$ must increase with $w$.

The sum of Country’s equilibrium payments to Creditors and default costs increases with $w$ because Country’s equilibrium probability of default given a policy failure increases with $w$ and because a default is more costly than a restructuring (under either a broad or narrow interpretation of the Clause). More specifically, observe that under an equal recovery equilibrium Creditors’ expected recovery, and therefore Country’s expected payment to Creditors in case of a policy failure, is $wP_{nd}(w)$. Because Country’s expected default costs are $w(1 - P_{nd}(w))$, Country’s total expected costs of a policy failure sum up to $w$.

5 Optimal Strength of a Pari Passu Clause

In this section, we present the trade-off associated with a higher probability of Country’s default given a policy failure as a result of a stronger Clause (higher $w$) and consider the effect of asymmetric information on the potential benefit of the Clause.

The potential value of a stronger clause lies in inducing a higher probability of Country’s default conditional on a policy failure on the equilibrium path or, if in equilibrium Country chooses a sure-to-succeed policy, off the equilibrium path. The higher prospect of default in turn increases Country’s total payments in case of a policy failure and thereby curbs Country’s incentives to choose too risky a policy.
If Country were certain to default in case of a policy failure, it would pay 1 if the policy either succeeds or fails (to Creditors in the former case and in default costs in the latter). Country would consequently fully internalize the risk of failure and choose the socially optimal policy. But unless the socially optimal policy is a sure-to-succeed one, making Country fully internalize the risk of failure is costly because Country incurs wasteful default costs all too often and Creditors obtain nothing if the policy fails.

To set up the welfare maximization problem associated with an optimal pari passu clause, let \( CR(w) \) and \( DC(w) \) denote, respectively, Creditors’ equilibrium recovery and Country’s equilibrium default costs given a policy failure as a function of \( w \). Country’s total payments given a policy failure as a function of \( w \) are then \( TF(w) \equiv CR(w) + DC(w) \).

Because Country pays Creditors 1 if the policy succeeds (with probability \( p \)) and its total payments given a policy failure are \( TF(w) \) (with probability \( 1 - p \)), Country’s interim (period-1) payoff is \( R(p) - [p + (1 - p)TF(w)] \), where \( R(p) \) is the policy payout. Differentiating with respect to \( p \), equating to zero, and rearranging, Country’s privately optimal (interior) policy in period 1 satisfies

\[
R'(p) = 1 - TF(w). \tag{5}
\]

Country thus equates the policy’s marginal payout (left-hand side) with the marginal cost of a safer policy, i.e., the difference between Country’s payments when the policy succeeds and when it fails (right-hand side).

Letting \( p(w) \) denote Country’s privately optimal policy as a function of \( w \), i.e., the value of \( p \) that satisfies (5) as an equality, Country’s welfare as a function of \( w \) is

\[
R(p(w)) - (1 - p(w))DC(w). \tag{6}
\]

The first term is the expected payout of policy \( p(w) \) and the second term is Country’s expected default costs as a function of \( w \) and \( p(w) \). It follows that an optimal pari passu clause minimizes the sum of (i) Country’s moral hazard costs and (ii) Country’s expected default costs.

More specifically, an optimal pari passu clause minimizes the sum of (i) \( R(\widehat{p}) - R(p(w)) \), where \( \widehat{p} \) denotes the first-best policy (moral hazard costs), and (ii) \( (1 - p(w))DC(w) \) (expected default costs). An optimal clause under which Country defaults with a strictly positive probability trades off (i) a lower payout difference between the first-best policy and Country’s privately optimal policy and (ii) a lower probability that Country’s policy fails and precipitates default against (iii) a higher probability that the Clause would be interpreted broadly given a policy failure and (iv) a higher probability of Country’s default given a broad interpretation of the clause (under an equal recovery equilibrium). Moral hazard costs decrease as Country’s total payments given a policy failure increase (up to 1). Consequently, any \( w \) that maximizes Country’s total payments in case of a policy failure for a given magnitude of default costs is potentially optimal.

The left diagram in Figure 2 shows Country’s total payments given a policy failure and the associated default costs as a function of \( w \) \( (TF(w)) \) for \( s = 1/2 \) and \( \alpha_{\min} = 1/2 \). For \( w \in (0, 1/2] \), a pari passu clause produces a full-participation equilibrium in which
Country’s total payments are $1/2$ and default costs are 0. For $w \in (1/2, w']$, where $w' \approx 0.809$, the Clause produces an equal recovery (unconstrained) equilibrium in which Country’s total payments are $w$ and default costs increase with $w$. For $w \in (w',1]$, the Clause produces a constrained equilibrium in which Country’s total payments increase with $w$ but Country’s expected default costs remain fixed. In this range, therefore, the only potentially-optimal $w$ is $1$.

To show the effect of asymmetric information on the potential benefit of a pari passu clause, we compare Country’s total payments and default costs given a policy failure for $s = 1/2$ versus $s = 2/5$ (again assuming $\alpha_{\text{min}} = 1/2$). The right diagram in Figure 2 shows a parametric plot of these quantities as $w$ goes from $s$ up to the value of $w$ for which the (equal recovery) equilibrium participation rate is $1/2$. Interestingly, Country’s total payments are higher under $s = 2/5$ when default costs are low and higher under $s = 1/2$ when default costs are high. A less transparent Country (with a higher $s$) might therefore be better able to constrain moral hazard through a pari passu clause than a more transparent one.

6 Extensions

6.1 A single holdout creditor

In this section, we assume there is a (potential) Single Holdout Creditor instead of a continuum of such creditors, while maintaining the assumption of a continuum of consenting creditors. A single holdout creditor and a continuum of competitive holdout creditors represent two tractable extremes. Real-life restructurings involve a few holdout creditors holding a large block of debt (as in Argentina’s restructuring). The concentration of holdout debt in a few hands in turn helps to reduce the legal and financial costs of challenging a restructuring plan.
The game unfolds as in our baseline model, except that if Country’s policy fails, Single Holdout Creditor may make a tender offer to buy a fraction \(1 - \alpha\) of Creditors’ debt at a price \(p\) and thereafter hold out his acquired debt; we shall refer to the fraction of holdout debt out of Country’s entire debt as the *holdout rate*.\(^{18}\)

**Definition 3 (restructuring equilibrium - single holdout creditor)** A restructuring equilibrium involving a Single Holdout Creditor is a triple of a participation rate \((\alpha)\), a restructuring demand \((d)\), and a tender offer price \((p)\) such that:

(i) the restructuring demand maximizes Consenting Creditors’ recovery given the participation rate;

(ii) given that a fraction \(1 - \alpha\) of Creditors tender their debt to Single Holdout Creditor at a price \(p\), no tendering creditor has incentive to consent rather than tender;

(iii) Single Holdout Creditor maximizes its profits by making a tender offer for a fraction \(1 - \alpha\) of Country’s debt at a price \(p\).

Consenting Creditors’ optimal restructuring demand as a function of the participation rate is identical to the one in the baseline case. Given that the tender offer succeeds, a tendering creditor has no incentives to consent rather than tender if and only if \(p\) is not lower than Consenting Creditors’ equilibrium recovery rate. Single Holdout Creditor maximizes its profits by acquiring a holdout block at which its marginal recovery is equal to Consenting Creditors’ recovery rate and by setting \(p\) equal to Consenting Creditors’ recovery rate.

The next proposition shows that the key results of the baseline case carry over to the single holdout creditor case.

**Proposition 4 (single holdout equilibria)** Suppose there is a Single Holdout Creditor and that the participation constraint is not binding.

(a) There exists a unique full-participation equilibrium iff \(w \in [0, s]\) (Area A in Figure 1).

(b) For \(w \in (s, 1]\) (Areas B + C in Figure 1), there exists a unique partial-participation equilibrium in which (i) the holdout rate increases with \(w\) and is strictly lower than under a continuum of holdout creditors, and (ii) for a sufficiently high degree of information asymmetry, Country’s probability of default and total payments given a policy failure increase with \(w\).

The proof of this proposition, along with a full characterization of the equilibrium outcomes, is relegated to an Online Appendix.

There does not exist a partial-participation equilibrium for \(w \in [0, s]\) because Single Holdout Creditor’s recovery is bounded above by \(w\), the probability that the Clause is interpreted broadly. Hence, Single Holdout Creditor cannot make a profitable tender offer.

\(^{18}\)In this and the following extensions, we assume that the participation constraint is not binding.
to acquire any fraction of Country’s debt that any creditor would accept. By contrast, if \( w \in (\tilde{w}, 1] \) Single Holdout Creditor’s marginal recovery on an infinitesimal holdout rate is \( w \), which is greater than \( \tilde{w} \). Consenting Creditors’ recovery rate under a full-participation equilibrium. Single Holdout Creditor can therefore make a profitable tender offer for a sufficiently small fraction of Country’s debt that induce creditors to tender.

The equilibrium holdout rate is strictly lower than under a continuum of dispersed holdout creditors, because Single Holdout Creditor internalizes the effect of a higher holdout rate on Country’s (higher) probability of default. To see this, observe that Single Holdout Creditor’s marginal recovery is lower than its average recovery and is equal (in equilibrium) to Consenting Creditors’ recovery rate. The holdout rate that equates Single Holdout Creditor’s marginal recovery and Consenting Creditors’ recovery rate is consequently lower than the holdout rate that equates Consenting Creditors’ and Holdout Creditors’ recovery rates under an equal recovery equilibrium. Finally, as in the baseline case and for equivalent reasons, Country’s probability of default and total payments given a policy failure both increase with \( w \) for a sufficiently high degree of information asymmetry.

6.2 An accumulating consenting creditor

We now consider the opposite case in which a (potential) Accumulating Consenting Creditor may make a tender offer for a fraction of Creditors’ debt and thereafter consent to a restructuring plan. If a creditor rejects Accumulating Consenting Creditor’s offer, he can either consent (along with Accumulating Consenting Creditor) or hold out. Recall that partial-participation equilibria in the baseline case do not maximize Creditors’ aggregate recovery given a policy failure. Our motivation is to inquire whether a tender offer by Accumulating Consenting Creditor can overcome the collective action problem of Creditors that leads to (ex post) inefficient outcomes in the baseline case. We conclude that it cannot.

The equilibrium conditions for Accumulating Consenting Creditor’s tender offer are, mutatis mutandis, the same as those for a single holdout creditor’s tender offer (optimal demand given participation rate, tendering creditors would not benefit from not tendering, Accumulating Consenting Creditor maximizes its net profits).

**Proposition 5 (single accumulating creditor)** For \( w \in (\tilde{w}, \bar{w}] \) (Area B in Figure 1), an Accumulating Consenting Creditor cannot make a profitable tender offer to buy a fraction \( \alpha \in (\alpha^*, 1] \) of Country’s debt that Creditors would accept, where \( \alpha^* \) denotes the participation rate under an equal recovery equilibrium.

Underlying Proposition 5 is a free-rider problem that obstructs a tender offer intended to eliminate or reduce holdout debt. For such a tender offer to be profitable for Accumulating Consenting Creditor, the tender offer price must not be higher than the post-acquisition recovery rate of consenting debt. However, for any \( \alpha \in (\alpha^*, 1] \), holdout debt yields a higher recovery rate than consenting debt. Thus, given that the tender offer succeeds, any tendering creditor would have incentive to hold out rather than consent. Accumulating
Consenting Creditor cannot therefore profitably acquire any fraction of Country’s debt that would preclude holdout debt.\footnote{19}{Because holdout debt yields a higher recovery rate than consenting debt for any $\alpha \in (\alpha^*, 1]$, the incentives not to tender here are stronger than the corresponding incentives in the Grossman and Hart’s (1980) takeover setup (where tendered and non-tendered shares have the same value). Mechanisms that can help to overcome the free-rider problem in takeovers, such as toeholds (Holmström and Nalebuff, 1992), may consequently fail here. Moreover, a freeze-out merger, which in practice resolves the Grossman–Hart problem (Amihud, Kahan, and Sundaram, 2014), is unavailable in our context.}

\section{6.3 Creditors’ recovery}

To simplify the analysis, we have treated the loan amount from Creditors to Country as a transfer that does not enter the welfare calculus. The amount Country is able to borrow from Creditors, however, has important welfare consequences, particularly when the debt proceeds serve Country’s growth and development needs. We now show that by reducing moral hazard, an optimal pari passu enables Country to borrow more from Creditors. Country thus obtains an additional benefit - other than a higher policy payout - from tying its hands through a pari passu clause.

\textbf{Proposition 6 (creditors’ recovery)} Under an optimal pari passu clause involving a strictly positive probability of Country’s default, Country is able to borrow more from Creditors than without a clause.

Country’s welfare consists of the policy payout less its expected default costs, whereas Country’s interim (period-1) payoff consists of Country’s welfare less its expected payments to Creditors (in period 2). Because in equilibrium Creditors break even, the difference between Country’s welfare and Country’s interim payoff is equal to the loan amount.

The reason Country is able to borrow more from Creditors under an optimal pari passu clause in which Country defaults with a strictly positive probability is that Country’s welfare is maximized under such a clause, whereas Country’s interim payoff decreases with the Clause strength, $w$. Country’s interim payoff decreases with $w$ - and therefore is lower under an optimal clause than without a clause - because Country is worse off midstream by having to pay more if the policy fails. More specifically, because the policy success payoff ($R(p) - p$) is independent of $w$ (other than through the effect of $w$ on $p$) whereas the policy failure payoff $(- (1-p)w)$ decreases with $w$, Country’s interim payoff for any choice of policy is higher under a lower $w$. Country’s payoff under the optimal policy given $w$ consequently decreases with $w$.\footnote{20}{By the Envelope Theorem, the derivative of Country’s period-1 payoff with respect to $w$ is $-(1-p) < 0$.}

\section{7 Conclusion}

This paper presented a sovereign debt model that explicates the strategic effects of pari passu clauses and their persistent ambiguous phrasing. We showed that an ambiguous
pari passu clause along with asymmetric information on a sovereign borrower’s ability to pay give rise to restructuring equilibria in which some creditors hold out and default is probabilistic. Under these equilibria, the sovereign’s probability of default and total payments in case of a policy failure increase with the probability that the pari passu clause will be interpreted in favor of holdout creditors. By varying the strength of the clause - i.e., the probability that it will be interpreted broadly - parties to a sovereign debt contract can implement an optimal (second-best) trade-off between reducing moral hazard and incurring dead-weight default costs.
Appendix

This Appendix proves Proposition 1 and Proposition 2.

We begin by introducing ancillary notations. Recall that Consenting Creditors’ demand is an “interior demand” if Country’s probability of default conditional on a broad interpretation of the Clause is strictly between 0 and 1 given Consenting Creditors’ participation rate ($\alpha$) and the degree of information asymmetry between Country and Creditors ($s$).

We define the set of Consenting Creditors’ potentially-optimal interior demands as

$$ D_i = \{d \in (0, \bar{d}] : P_{nd}(\alpha, d, s) \in (0, 1)\}. \quad (A1) $$

That is, $D_i$ consists of all strictly positive restructuring demands (weakly) less than $\bar{d}$ for which Country’s probability of default is strictly between 0 and 1.

The following definition denotes the infimum and supremum of a non-empty $D_i$.

**Definition A1** (i) $\underline{d}(\alpha, s) \equiv \max\{0, \bar{d} - (1 - \alpha)\} = \inf(D_i : D_i \neq \emptyset)$; and (ii) $\bar{d}(\alpha, s) \equiv \min\{\bar{d} - (1 - \alpha), \bar{d}\} = \sup(D_i : D_i \neq \emptyset)$.

The infimum of a non-empty $D_i$ as a function of $\alpha$ and $s$ is the maximum of (i) the maximum no-default demand ($\bar{d} - (1 - \alpha)$) and (ii) 0. The corresponding supremum is the maximum of (i) the minimum default demand ($\bar{d} - (1 - \alpha)$) and (ii) $\bar{d}$.

The next definition denotes the infimum and supremum of participation rates ($\alpha$) for which $D_i$ includes $\bar{d}$ or is empty.

**Definition A2** (i) $\underline{\alpha}(s) \equiv 1 - s = \inf(\alpha : \bar{d} \in D_i)$; and (ii) $\bar{\alpha}(s) \equiv 1 - \bar{s} = \inf(\alpha : D_i \neq \emptyset)$.

Underlying the infimum and supremum in Definition A2 is the fact that Country’s probability of default given a broad interpretation of the Clause is strictly positive iff the sum of Creditors’ demands is strictly less than the upper bound of the support of Country’s payout distribution ($P_{nd}(\alpha, d, s) > 0$ iff $d + (1 - \alpha) < \bar{\sigma}$). If $\alpha > \bar{\alpha}(s) \equiv 1 - s$, Holdout Creditors’ demand is strictly less than $s$ ($1 - \alpha < s$) and therefore the sum of Creditors’ demands is strictly less than $\bar{s}$ for any $d \leq \bar{d}$. If $\alpha \leq \underline{\alpha}(s) \equiv 1 - \bar{s}$, Holdout Creditors’ demand is greater than or equal to $\bar{s}$ ($1 - \alpha \geq \bar{s}$) and so is the sum of Creditors’ demands for any $d > 0$.\(^{21}\)

Now, for $d \in D_i$ the sum of Creditors’ demands lies in the interval $(\bar{s}, \bar{d})$. Writing $\alpha - \underline{\alpha}$ for $\bar{s} - (1 - \alpha)$, subtracting $d$, and dividing the difference by $s$, Country’s probability of no default given a broad interpretation of the Clause is

$$ P_{nd}(\alpha, d, s) = (1/s) \times (\alpha - \underline{\alpha}(s) - d). \quad (A2) $$

\(^{21}\)Note that $\bar{d} \in (\underline{\alpha}(s), \bar{\alpha}(s))$.  

21
We now turn to the proof of Proposition 1.

**Proposition 1 (full-participation equilibrium)** There exists a payoff dominant full-participation equilibrium if and only if \( w \in [0, \bar{s}] \) (Area A in Figure 1).

**Proof.** Sufficiency follows from the text. To show necessity, suppose that \( w \in (\bar{s}, 1] \) and that all Creditors participate and make a demand of \( \bar{s} \), which Country accepts. Creditors accordingly obtain a recovery rate of \( \bar{s} \). If a sufficiently small mass of Creditors deviate to holding out, the participation rate will decrease to \( 1 - \varepsilon \). By \((A2)\), the probability that Country does not default given a broad interpretation of the Clause would then be

\[
P_{nd}(1 - \varepsilon, \bar{s}, s) = \frac{1 - \alpha(s) - \bar{s}}{1 - \varepsilon/s},
\]

which for any \( w \in (\bar{s}, 1] \) and a sufficiently small \( \varepsilon \) is greater than the putative equilibrium recovery rate of \( \bar{s} \). For \( w \in (\bar{s}, 1] \) there are therefore incentives to hold out, which frustrate a full-participation equilibrium.

To prove Proposition 2, we shall prove the following, more comprehensive proposition.

**Proposition 2A (equal recovery equilibria)** Let

\[
\bar{w}(s) \equiv s \cdot \left(\frac{5/4 - \sqrt{s^2 + (\bar{\pi}(s)/2)^2}}{s}\right)^{-1}
\]

for \( s \in (\bar{s}, 1/2] \) and \( \bar{w}(s) \equiv \max\{s/(1 - \bar{s}), \bar{s}\} \) (see dashed and blue curves in Figure 1) and assume that the participation constraint is not binding.

(a) For \( w \in (\bar{s}, \bar{w}] \) (Area B in Figure 1), there exists a unique equal recovery equilibrium where the participation rate and Consenting Creditors’ restructuring demand as a function of the Clause’s strength \( (w) \) and the information asymmetry between Country and Creditors \( (s) \) are:

\[
(\alpha^*(w, s), \ d^*(w, s) =
\begin{cases}
\left( \bar{s} + (A - B)/2 + \sqrt{\bar{s}A + ((A - B)/2)^2}, \bar{s} \right) & \text{if } w \in (\bar{s}, \bar{w}]
\left( 2A + \sqrt{(2A)^2 + B^2} - (1/2) \times \left( 2A + B + \sqrt{(2A)^2 + B^2} \right) \right) & \text{if } w \in (w, \bar{w}],
\end{cases}
\]

where \( A(w, s) \equiv s(1 - w)/w \) and \( B(w, s) \equiv s(1 - w)/w - \alpha(s) \).

The equilibrium participation rate, restructuring demand, and Country’s probability of no-default decrease with \( w \) (strictly, weakly, and strictly, respectively) and are bounded below by \( 4\alpha(s), 2\alpha(s), \) and \( \alpha(s)/s \), respectively.

(b) For \( w \in (\bar{w}, 1] \) (Area C in Figure 1) there does not exist an equal recovery equilibrium.

**Proof.** We proceed by deriving the equal recovery participation rate as a function of Consenting Creditors’ restructuring demand and then deriving Consenting Creditors’ optimal restructuring demand as a function of the participation rate. Using the two equilibrium conditions, we solve for the equal recovery equilibrium participation rate and
restructuring demand as a function of \( w \) and \( s \). To show that there does not exist an equal recovery equilibrium, we find conditions on \( w \) and \( s \) such that there does not exist an interior demand that maximizes Consenting Creditors’ recovery and satisfies the equal recovery condition for any participation rate.

We begin by deriving the equal recovery participation rate as a function of the restructuring (interior) demand. Rearranging terms in the equal recovery condition in (4) gives \( P_{nd}(\alpha, d, s) = d/(\alpha - d) \times (1 - w)/w \). Plugging in \( ((\alpha - \alpha(s)) - d)/s \) for \( P_{nd}(\alpha, d, s) \) (from (A2)) and solving for \( \alpha \) we get

\[
\hat{\alpha}(d, w, s) = d + \alpha(s)/2 + \sqrt{d \cdot s \cdot (1 - w)/w + \alpha(s)/2}^2, \tag{A3}
\]

for \( d \in D_i \). The condition (A3) thus restates the equal-recovery equilibrium condition by associating with any interior demand an equal-recovery participation rate.

Before proceeding, we derive Consenting Creditors’ marginal benefit and cost of making a higher interior demand. Recall from (3) that Consenting Creditor choose a restructuring demand to maximize \( d \times [(1 - w) + wP_{nd}(\alpha, d, s)] \). Consenting Creditors’ marginal benefit and cost of making a higher interior demand are accordingly

\[
MB_d(\alpha, d, w, s) \equiv 1 - w + wP_{nd}(\alpha, d, s) \tag{A4a}
\]

and

\[
MC_d(\alpha, d, w, s) \equiv -wd \cdot \partial P_{nd}(\alpha, d, s)/\partial d. \tag{A4b}
\]

The marginal benefit of a higher interior demand (A4a) is the probability that the Clause is either (i) interpreted narrowly or (ii) interpreted broadly and Country does not default. The corresponding marginal cost (A4b) is the demand times the marginal decrease in the probability that Country does not default, i.e., the probability that the Clause is interpreted broadly times the marginal decrease in the probability that Country can pay both Consenting and Holdout Creditors. Note that Consenting Creditors’ expected recovery is \( d \times MB_d(\alpha, d, w, s) \).

We now turn to deriving Consenting Creditors’ optimal interior demand as a function of the participation rate.

Case I: \( d^* = \bar{s} \). Given a participation rate \( \alpha \), Consenting Creditors maximize their expected recovery by making a restructuring demand \( d = \bar{s} \) iff:

\[
MB_d(\alpha, d, w, s) > MC_d(\alpha, d, w, s) \quad \text{for all } d \in (d(\alpha, s), \bar{s}) \tag{A5a}
\]

subject to \( \alpha > \alpha(s) \).

The inequality condition (A5a) ensures that Consenting Creditors’ marginal benefit of increasing any interior demand lower than \( \bar{s} \) is greater than the corresponding marginal cost. The inequality condition (A5b) ensures that a demand of \( \bar{s} \) is an interior demand (if \( \alpha \leq \alpha(s) \) a demand of \( \bar{s} \) is a default demand by definition of \( \alpha(s) \)). For values of \( \alpha \) that satisfy (A5b), therefore, a no-default demand yields a lower recovery than an interior demand of \( \bar{s} \).
Now, because Consenting Creditors’ marginal benefit of making a higher interior demand decreases with \( d \) and the corresponding marginal cost increases with \( d \), an interior demand of \( s \) is optimal iff \( \lim_{d \to s} MB_d(\alpha, d, w, s) \geq \lim_{d \to s} MC_d(\alpha, d, w, s) \). Plugging in \( ((\alpha - \alpha(s)) - d)/s \) for \( P_{nd}(\alpha, d, s) \), \(-1/s\) for \( \partial P_{nd}/\partial d \), and \( \bar{s} \) for \( d \) in the previous inequality (because both limits involve continuous functions) we get

\[
\alpha > \alpha_m(w, s),
\tag{A6}
\]

where \( \alpha_m(w, s) \equiv \overline{\alpha}(s) + \bar{s} - s/w > \overline{\alpha}(s) \) because \( w > 1/2 \geq \bar{s}/s \).

The equal-recovery equilibrium participation rate is obtained by plugging in \( \bar{s} \) for \( d \) in (A3). Substituting the equal-recovery participation rate for \( \alpha \) and solving for \( w \) that satisfies (A6) gives \( w \leq \bar{w}(s) \), where \( \bar{w}(s) > \bar{s} \) for \( s > \bar{s} \equiv 7/2 - \sqrt{10} \approx 0.33 \).

**Case II:** \( d^* < \bar{s} \). Given a participation rate \( \alpha \), Consenting Creditors maximize their expected recovery by making a restructuring demand iff for \( \overline{d} \in (d(\alpha, s), \bar{d}(\alpha, s)) \)

\[
\begin{align*}
MB_d(\alpha, \widehat{d}, w, s) &= MC_d(\alpha, \widehat{d}, w, s) \\
\widehat{d} \times MB_d(\alpha, \widehat{d}, w, s) &\geq (1 - w) \min\{\alpha, \bar{s}\} \quad \text{for} \quad \alpha \leq \overline{\alpha}(s).
\end{align*}
\tag{A7}
\]

The two conditions in (A7) together ensure that Consenting Creditors’ marginal benefit of increasing their demand is equal to the corresponding marginal cost (equality condition)\(^{22}\) and that Consenting Creditors’ expected recovery is higher under an optimal interior demand than under a default demand (inequality condition).\(^{23}\)

Plugging in \( ((\alpha - \alpha(s)) - d)/s \) for \( P_{nd}(\alpha, d, s) \) and \(-1/s\) for \( \partial P_{nd}/\partial d \) in the equality condition in (A7) and solving for \( \overline{d} \) we get

\[
\widehat{d}(\alpha, w, s) = (\alpha + B(w, s))/2,
\tag{A8}
\]

where \( B(w, s) = s(1 - w)/w - \alpha(s) \). (A8) is Consenting Creditors’ candidate optimal interior demand as a function of the participation rate for such values of \( \alpha \) for which Consenting Creditors’ demand is an interior demand. Solving for \( \alpha \) for which \( \widehat{d}(\alpha, w, s) \in (d(\alpha, s), \bar{d}(\alpha, s)) \) gives \( \alpha \in (\alpha_l, \min\{\alpha_m, \alpha_{nd}\}) \), where \( \alpha_m(w, s) \) is defined in (A6), \( \alpha_{nd}(w, s) \equiv \alpha(s) + s(1 + 1/w) \) is the value of \( \alpha \) for which \( \widehat{d}(\alpha, w, s) = \bar{s} - (1 - \alpha) \) (maximum no-default demand), and \( \alpha_l(w, s) = \alpha(s) + s(1 - 1/w) \) is the value of \( \alpha \) for which \( \widehat{d}(\alpha, w, s) = \bar{s} - (1 - \alpha) \) (minimum default demand).

Now, the right-hand side of the inequality condition in (A7) increases with \( \alpha \), whereas the left hand side is capped at \( (1 - w)\bar{s} \). It therefore suffices to consider the case where \( \min\{\alpha, \bar{s}\} = \alpha \). Plugging in \( d/(\alpha - d) \times (1 - w)/w \) for \( P_{nd}(\alpha, d, s) \) (by the equal-recovery condition), the inequality \( d \times MB_d(\alpha, d, w, s) \geq (1 - w)\alpha \) reduces to \( d \geq \alpha/2 \). But the right-hand side of (A8) is greater than or equal to \( \alpha/2 \) iff \( B(w, s) \geq 0 \). The maximum value of \( w \) for which Consenting Creditors’ optimal demand is an equal-recovery interior

\(^{22}\)The second-order condition for a maximum is satisfied because the second derivative of Consenting Creditors’ expected recovery is \(-1/s < 0\).

\(^{23}\)We assume that Consenting Creditors will make an interior demand if they are indifferent between making such a demand and making a default demand.
demand (rather than a default demand) is therefore the value of $w$ that satisfies $B(w, s) = 0$, which value is $s/(1 - \hat{s}) > / \leq \hat{s}$ for $s > / \leq \hat{s} \equiv \sqrt{3} - 3/2 \approx 0.23$. It follows that for (i) $s > \hat{s}$ and $w \in (s/(1 - \hat{s}), 1]$ or (ii) $s \leq \hat{s}$ and any $w \in (\hat{s}, 1]$ there does not exist an interior demand that maximizes Consenting Creditors recovery and satisfies the equal recovery condition for any participation rate.

To obtain the equilibrium participation rate, we substitute $\hat{d}(\alpha, w, s)$ for $d$ in $(A3)$ and solve for $\alpha$. Plugging the equilibrium participation rate back into $\alpha$ in $(A8)$ gives the equilibrium restructuring demand.\(^{24}\)

The lower bounds of the equilibrium participation rate, restructuring demand, and probability of no-default are obtained by plugging in 0 for $B$ and $\alpha$ for $A$ in the respective equilibrium expressions.

We conclude with an example that illustrates the equilibrium outcome for $s = 1/2$.

**Example A1** Suppose that $s = 1/2$ and that the participation constraint is not binding. There exists a unique equal recovery equilibrium where the participation rate and Consenting Creditors’ restructuring demand as a function of the Clause’s strength ($w$) are:

$$(\alpha^*(w), d^*(w)) = \begin{cases} 
(1/2 + \sqrt{A/2}, 1/2) & \text{if } w \in (1/2, c] \\
(2A + 5A^2, (1/2) \times (3A + 5A^2)) & \text{if } w \in (c, 1],
\end{cases}$$

where $A(w) \equiv (1 - w)/(2w)$ and $c \equiv (1 + 1/\sqrt{5})/2 \approx 0.72$.

When $s = 1/2$, $A(w) = B(w) = (1 - w)/(2w)$ (because $\alpha(1/2) \equiv 1 - \bar{s} = 1 - 1 = 0$). Plugging in $A$ for $B$ and $(1 - w)/(2w)$ for $A$ in the equilibrium expressions in Proposition 2A gives the expressions in the Example. The Example illustrates that as $w$ increases, the drop in the equilibrium participation rate is greater than the corresponding drop in Consenting Creditors’ restructuring demand and therefore the sum of Creditors’ demands increases with $w$. In particular, for low values of $w$, the equilibrium restructuring demand remains the same as $w$ increases while the equilibrium participation rate decreases with $w$; for high value of $w$, by contrast, both the equilibrium participation rates and restructuring demand decreases with $w$, but the equilibrium participation rate drops more steeply with $w$ than the equilibrium restructuring demand. Furthermore, because $d^*(w) \geq \alpha^*(w)/2$ for any $w \in (1/2, 1]$, there exists an equal recovery equilibrium for any such $w$.

\(^{24}\)For $s \in (\hat{s}, \bar{s})$, both the equilibrium restructuring demand and participation rate drop *discontinuously* at $w = \hat{s}$. The discontinuous drop results from the fact that for high participation rates CC’s optimal demand is a no-default demand under which HC’s recovery rate is higher than CC’s recovery rate. HC’s higher recovery rate produces a pressure to hold out, which lowers the participation rate and restructuring demand until CC’s optimal demand becomes an interior demand. The equilibrium probability of no-default, by contrast, decreases *continuously* with $w$. 

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References


NML Capital, Ltd. v. Argentina, 699 F.3d 246 (2nd Cir. 2012).


